Fall 2013 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. In a square pyramid, the sides of the square base are doubled and the height is halved. By what percent is the volume of the original pyramid changed?

2. Ashley's creature box for her science experiment contains centipedes and spiders. Despite the name 'centipede'each of Ashley's centipedes has 30 legs. She counts a total of 23 insects and 228 legs. How many centipedes does Ashley have? By the way, spiders have 8 legs!

3. Given that $b > a$, write down the least of these numbers:
$$\frac{4b+a}{5}, \qquad \frac{3a+7b}{10}, \qquad \frac{a+8b}{9}, \qquad \frac{4b+3a}{7}, \qquad \frac{5a+5b}{10}$$

4. The sum of the supplements of the angles of a triangle always equals what? Answer in degrees.

5. Find the area of the region $T = \{(x, y) : |x| + 3|y| \leq 4\}$.

6. Find the integer m such that
$$(\sqrt{2}-1)^4 = \sqrt{m} - \sqrt{m-1}$$

7. Segment AD is an altitude of equilateral triangle ABC and segment DE is an altitude of triangle CDA. Find the ratio AE/EC.

8. Determine how many ordered pairs of positive integers (a, b) satisfy $\frac{1}{a} + \frac{2}{b} = \frac{3}{7}$.

9. In quadrilateral $ABCD$, $\angle BAC = 30^\circ$, $\angle CAD = 50^\circ$, and $BA = CA = DA$. Find the measurement of $\angle BDA$ in degrees.

10. A point (a, b) is first reflected across the x-axis, then across the line $y = -x$ and finally across the origin to land at the point $(12, 5)$. Find the sum $a + b$.

11. In how many different ways can 10 identical chairs be distributed to 4 distinct tables? A table may be left without any chairs at all. Answer in standard integer form.

12. In $\triangle ABC$ we have $AB = 13$, $AC = 15$, and $BC = 14$. Find the length of the median from vertex A to side BC.

13. If the point (x, y) satisfies
$$x^3 - 71x = y^3 - 71y$$
but does not satifsy $x = y$ then what is the value of $x^2 + xy + y^2$?

14. Point A has coordinates $(19, -104/3)$ while point B has coordinates $(-43/3, -14)$. Find the coordinates of a point C which lies on the perpendicular bisector of segment AB given that both of the coordinates of C are integers.

15. Let $f(x, y) = yx^2 - (2y + 1)x + y$. Solve $f(x, 6) = 0$.