## Spring 2014 GDCTM/McNabb Algebra Two Contest

## NO Calculators Allowed

1. Point $P$ can be any point on a fixed circle of radius seven, while point $Q$ can be any point in the plane of the circle at a distance of three from point $P$. Find the area of the set of all possible locations of the point $Q$.
2. A baseball is hit so that its height in feet $t$ seconds after impact is given by $h(t)=80 t-16 t^{2}+3$. What is the maximum height, in feet, this baseball reaches?
3. Find the smallest positive integer greater than 200 with exactly 8 positive factors.
4. If $\sec x-\tan x=2$ find the value of $\sec x+\tan x$.
5. Four apples are weighed 2 at a time in all possible ways, giving these six weights in ounces: $16,16,18,19,21$, and 21 . How many ounces do all four apples together weigh?
6. Factor $x^{4}+2 x^{3}-15 x^{2}+8 x-1$ into the product of two quadratic polynomials with integer coefecients.
7. An airplane makes a round trip flying the same route each way. Its speed (relative to the ground) is 480 mph in one direction while it is 600 mph in the other. What is the plane's average speed for the entire round trip?
8. How many ordered pairs of positive integers $(x, y)$ satisfy the equation

$$
x y+x+y=2014
$$

?
9. If $\frac{(a-b)(c-d)}{(b-c)(d-a)}=4$ then find the value of $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.
10. Eight cows graze a pristine field bare in 40 days. It would take 15 cows just 12 days to graze the same pristine field bare. How many days would it take 10 cows to graze that same pristine field bare? Assume that the grass in this field grows at a constant rate and the cows graze at a constant rate.
11. Find the sum of the square roots of the roots of the quadratic equation $x^{2}-39 x+25=0$.
12. In a right triangle, the sum of the legs is equal to 10 and the altitude from the right angle to the hypotenuse has length 3 . Find the length of the hypotenuse.
13. If $x \ln x=15$ then what is the least integer greater than $x$ ?
14. If $\log _{a} b=2$ and $\log _{b} c=4$, find the value of $\log _{c} a$.
15. Solve the equation

$$
\sqrt[3]{x+8}+\sqrt[3]{8-x}=1
$$

