Fall 2016 McNabb GDCTM Contest
Algebra Two

NO Calculators Allowed

1. How many two-digit integers are divisible by five?

2. Ruprecht is running North at a certain constant rate, while on the same road his friend Waldo is running South toward him at a constant rate that is 4 mph faster than one-third of Ruprecht's rate. If they started 40 miles apart and it took 2 hours and 15 minutes for them to meet, what is Waldo's rate?

3. Find the sum of all numbers which are two greater than their own reciprocal.

4. How many distinct numbers can be written in the form a/b, where $a \in \{1, 2, 3, 4, 5\}$ and $b \in \{5, 6, 7, 8, 9\}$?

5. Simplify to standard complex number form:

$$\frac{1}{i} + \frac{2}{i+1}$$

6. Find the coefficient of x^6 in the product $P(x) \cdot Q(x) \cdot R(x)$ where

$$P(x) = (x+1)(x^4 + 2x^3 + 4x^2 + 8x + 16)$$
$$Q(x) = (x-1)(x^2 - x - 1)$$
$$R(x) = (x-2)(x^2 + x + 1)$$

7. Let $\lfloor x \rfloor = $ the greatest integer $\leq x$. FInd all solutions to the equation

$$3x - 2\lfloor x \rfloor = 2$$

8. Find the sum of the reciprocals of the positive divisors of 1000.

9. In a magical forest, the cylindrical bamboo stalks increase their height at a rate of 4 inches per day. Their radius stays a constant value of $1/\sqrt{\pi}$ inch. One stalk of bamboo grows in every other (checkerboard-style) 3 inch by 3 inch square of a 10 foot by 10 foot plot of the forest. One day a panda comes across this plot of bamboo. On that day the bamboo started at a height of 4 feet. Due to the panda's considerable appetite, the rate of growth of the bamboo stalks' height is cut in half. In how many days will the volume of bamboo in this plot first reach a total volume of 50,000 cubic inches? Count the day the panda arrives.

10. In how many ways can one arrange the letters of WINTER in such a way that the two vowels are never adjacent?

11. A particle starts at a fixed point P on a circle with center C. The particle moves on the circle, either 45° counterclockwise or 45° clockwise, each with probability 1/2. After eight such moves, the particle is at a point we call Q. What is the probability that $\angle PCQ$ is right? (Note: it is possible that points P and Q coincide.)

12. Calculate the remainder when 100^{1008} is divided by 2017.

13. Simplify

$$\sqrt[3]{99 - 70\sqrt{2}}$$

14. Suppose that the roots of the polynomial $x^3 + 2x^2 + 3x - 7$ are a, b, and c. Find a monic polynomial with coeffiecients in $\mathbb{Z}$ with roots ab, bc, and ac.

15. Factor completely over the integers:

$$x^8 + x^5 + x^3 + 1$$