

FALL 2016 McNABB GDCTM CONTEST
CALCULUS

NO Calculators Allowed

Assume all variables are real unless otherwise stated in the problem.

1. Simplify to standard complex number form:

$$\frac{1}{i} + \frac{2}{i+1}$$

2. How many positive integers have a length of four digits in both base seven and base eight?
3. There exists a polynomial $Q(x)$ with integer coefficients such that

$$x^{12} + x^9 + x^6 + x^3 + 1 = Q(x) \cdot (x^4 + x^3 + x^2 + x + 1)$$

Find the sum of the coefficients of $Q(x)$.

4. Bill has a magic bag containing red and blue stones. This bag has an interesting property—whenever a stone is drawn, two stones of the other color appear in the bag! If Bill starts out with six red and four blue stones in the bag, what is the probability that his first four draws from the bag are all red? Note that the stones that Bill draws out are not put back in the bag.
5. Let $f(x) = \frac{3x+8}{2x+5}$ and $g(x) = \frac{3+2x}{3x}$. Find the value of $(f \circ g^{-1})^{-1}\left(\frac{11}{7}\right)$.
6. The value $\sin(75^\circ)$ can be expressed in the form

$$\frac{\sqrt{a} + \sqrt{b}}{c}$$

where a , b , and c are positive integers, and a and b have no perfect square factors greater than one. Find the product abc .

7. Randomly and independently chose two real numbers a and b from the interval $(0, 1)$. What is the probability that $ab \leq 3/8$?
8. How many real roots does $f'(x)$ have given that

$$f(x) = x(x+1)(x+2)(x+3)(x+4)$$

?

9. Let $n \in \mathbb{N}$. Find the n^{th} derivative of $f(x) = x^n e^x$ at $x = 0$.
10. Consider the function

$$f(x) = \begin{cases} 1+x & x \in \mathbb{Q} \\ e^x & x \notin \mathbb{Q} \end{cases}$$

What is its derivative at $x = 0$?

11. A rocket is launched vertically at a constant speed of 250 meters per second. A camera is located 2000 meters horizontally from the launch pad. As soon as the rocket launches, the camera moves horizontally away from the launch pad at a rate of 50 meters per second. The camera is tracking the rocket and measuring the angle of elevation θ . Find $d\theta/dt$ when the rocket's altitude is 6000 meters.
12. Find the maximum possible value of y if $x^2 + xy + y^2 = 300$
13. Determine
- $$\lim_{x \rightarrow \infty} \left(x^2 - x^4 \ln \left(\frac{1 + x^2}{x^2} \right) \right)$$
14. Let $f(x) = x^2 + 3x$ for $-1 \leq x \leq 1$. Let $g(x) = f^{-1}(x)$. Find the value of $g''(0)$.
15. At which x -coordinate does the graph of $y = e^x$ bend the most? For example, a parabola bends the most at its vertex. While an ellipse bends the most at the endpoints of its major axis.