FALL 2010 MCNABB GDCTM CONTEST
ALGEBRA I

NO Calculators Allowed

1. The algebraic expression

$$(a - b - c) - (a + b - c) - (a - b + c) - (-a + b - c)$$

is equivalent to

(A) $a + b + c$ (B) $-a$ (C) $-2a$ (D) $-2b$ (E) $b - c$

2. An automobile goes $y/9$ yards in d seconds. How many feet does it travel in two minutes time?

(A) $\dfrac{40y}{d}$ (B) $\dfrac{40d}{3y}$ (C) $\dfrac{3y}{40d}$ (D) $120yd$ (E) $\dfrac{120y}{d}$

3. The well-known formula $f = (9/5)c + 32$ relates the temperature f in Fahrenheit to the temperature c in Celsius. For how many values of f satisfying $32 \leq f \leq 212$, will the temperature be an integer in both of these scales?

(A) 9   (B) 10   (C) 19   (D) 20   (E) 21

4. Four circles are drawn, all in the same plane. Find the maximum number of regions they can form. The diagram shows how four circles may form 8 regions.
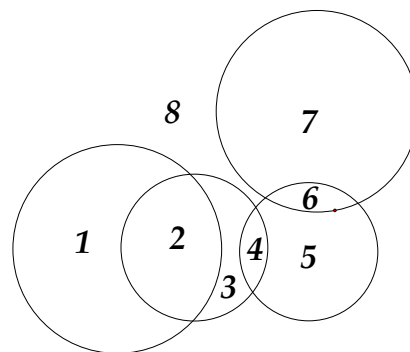
(A) 12   (B) 13   (C) 14   (D) 15   (E) 16

5. If the square root of a positive number falls between seven and eight, then the cube root of this number must fall between

    (A) 7 and 8   **(B)** 6 and 7   **(C)** 5 and 6

    (D) 4 and 5   **(E)** 3 and 4

6. John has 54 coins totaling one dollar in value. Some are pennies; some are nickels; some are dimes. He has no other kind of coin. How many nickels does John have?

    (A) 7   **(B)** 9   **(C)** 11   **(D)** 13   **(E)** 15

7. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

    (A) 124   **(B)** 241   **(C)** 248   **(D)** 428   **(E)** 503

8. On a certain island, there are currently 1000 inhabitants, and 91% of these inhabitants were born there. Then some of these native inhabitants leave, so that now only 90% of the inhabitants of the island were born there. Assuming no other kind of change (births, deaths, immigration, etc...) in the population took place, how many of the native inhabitants left?

    (A) 9   **(B)** 10   **(C)** 40   **(D)** 90   **(E)** 100

9. How many positive integers less than 200 have an odd number of factors?

    (A) 6   **(B)** 7   **(C)** 8   **(D)** 9   **(E)** 14

10. At Zeke's Zucchini Stand, 3 zucchini's and 2 squash cost $4.75, while 2 zucchini's and 3 squash cost $5.25. How much would 3 zucchini's and 3 squash cost?

    (A) $5.50   **(B)** $5.75   **(C)** $6   **(D)** $6.25   **(E)** $6.50

11. Two sides of a right triangle have lengths 6 and 8 respectively. The product of all the possible lengths of the third side can be written in the form $\sqrt{N}$, for some integer N. What is N?

    (A) 100   **(B)** 2400   **(C)** 2800   **(D)** 3200   **(E)** 3600

12. In how many ways can the the letters in *syzygy* be arranged so that the three y's do not all occur together?

    (A) 96   **(B)** 112   **(C)** 113   **(D)** 114   **(E)** 120

13. Points A, B, C, and D are collinear and occur in the same order as given. If the ratio $AB : BC$ equals 3 and the ratio $BD : AB$ equals 8/3, then determine the ratio $CD : AC$.

    (A) $\dfrac{5}{3}$ **(B)** $\dfrac{3}{2}$ **(C)** 2   **(D)** $\dfrac{7}{4}$ **(E)** $\dfrac{8}{3}$

14. A baseball team has won 50 games out of 75 so far played. If there are 45 games yet to be played, how many of these must be won in order for the team to finish its season having won exactly 60% of its games?

    (A) 20   **(B)** 21   **(C)** 22   **(D)** 23   **(E)** 72

15. A map maker has four colors available to color this map consisting of 5 counties. Each county is colored with a single color. No two counties that share a common boundary may be colored the same. In how many ways can our map maker color this map?

    (A) 36   **(B)** 48   **(C)** 72   **(D)** 96   **(E)** 120
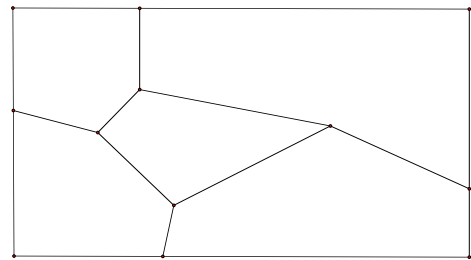
16. The value of the fraction

$$\frac{3+6+9+\cdots+99}{4+8+12+\cdots+132}$$

is

(A) 2/3   (B) 3/4   (C) 4/5   (D) 5/6   (E) 7

17. A bag contains 4 quarters and 2 dimes. If 3 coins are randomly removed from the bag, what is the expected total value in cents of these three coins?

(A) 50   (B) 55   (C) 60   (D) 65   (E) 75

18. The midpoints of the sides of a triangle are $(7, 4)$, $(1, 2)$, and $(1, 6)$. What is the area of this triangle?

(A) 12   (B) 24   (C) 30   (D) 36   (E) 48

19. The set S contains seven numbers whose mean is 202. The mean of the four smallest numbers in S equals 100, while the mean of the four largest numbers in S equals 300. What is the median of all the numbers in S?

(A) 184   (B) 186   (C) 192   (D) 196   (E) 200

20. To specify the order of operations in multiplying 5 numbers together, three sets of parentheses are needed. Two ways, for example, are $((ab)(cd))e$ and $(((ab)c)d)e$. In how many ways can these three sets of parentheses be arranged? Assume the order of the numbers a through e is never changed.

(A) 14   (B) 15   (C) 16   (D) 17   (E) 18

21. Let $f(x) = |x+1| - |x| + |x-1|$. What is the minimum value of $f(x)$?

(A) -2   (B) -1   (C) 0   (D) 1   (E) 2

22. Let a, b, and c be positive integers with $LCM(a,b) = 48$, $LCM(b,c) = 42$, and $LCM(c,a) = 112$ then the value of $LCM(a,b,c)$ is

(A) 224   (B) 336   (C) 448   (D) 672   (E) cannot be determined

23. Find the area of quadrilateral $ABCD$ given that $AB = 13$, $BC = 10$, $CD = 10$, $DA = 13$, and $AC = 13$.

(A) 80   (B) 90   (C) 100   (D) 110   (E) 120

24. While Xerxes marched on Greece his army streched out for 50 miles. A dispatch rider had to ride from the rear to the head of the army, then instantly turn about and return to the rear. While he did this, the army advanced 50 miles. How many miles did the rider ride?

(A) 100   (B) 150   (C) $50 + 100\sqrt{2}$ (D) $100\sqrt{2}$ (E) $50 + 50\sqrt{2}$

25. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32   (B) 28   (C) 24   (D) 20   (E) 16