SPRING 2014 GDCTM/McNABB ALGEBRA ONE CONTEST

NO Calculators Allowed

1. In how many way can the letters in $DALLAS$ be arranged? Include the original way!

2. Jessica has three standard six-sided dice—one is blue, one is red, and one is white. In how many ways can she roll the dice so that the sum of the dots showing on top equals 12?

3. Let a, b, and c be distinct positive integers such that at least two of them are divisible by 6 and at least two of them are divisible by 9. Find the minimum possible value of $a + b + c$.

4. Find the number of factors of 30! which are perfect cubes.

5. How many positive integers less than 20 can be written as the sum of three consecutive integers?

6. For what value of the constant a does the equation $ax - 3a + 1 = 4(3x - a) - ax$ have no solution?

7. How many times do the graphs of the functions $f(x) = 1000|x|$ and $g(x) = x^2/1000$ intersect?

8. Find all value(s) of the constant a so that the line $ax + 2y = 7$ is perpendicular to the line $ax - 3y = 2$.

9. Find the prime factorization of 9991.

10. Four cards are drawn randomly from a standard 52 card deck. What is the probability that no two of these cards belong to the same suit?

11. Hezy needs to buy 8 apples, 6 oranges, and 3 plums. He recalls that the prices per piece of these fruits are 27 cents, 72 cents, and 54 cents, but he does not remember which price goes with which fruit. What is the least amount in cents that Hezy must bring with him to the store to ensure that he can buy all the fruit he needs?

12. A running back tries to first elude tackler A and then tackler B. His probability of eluding A is $2/3$ and his probability of eluding both A and B is $1/2$. What is the probability that the running back eludes B once he has eluded A?

13. Find the sum of the square roots of the roots of the quadratic equation $x^2 - 39x + 25 = 0$.

14. Solve the system

$$ab^2c^2 = -18$$
$$a^3bc^2 = 12$$
$$ab^2c = -36$$

15. Eight cows graze a pristine field bare in 40 days. It would take 15 cows just 12 days to graze the same pristine field bare. How many days would it take 10 cows to graze that same pristine field bare? Assume that the grass in this field grows at a constant rate and the cows graze at a constant rate.