Spring 2019 McNabb GDCTM Contest
Algebra One

No calculators are allowed. You have 45 minutes. Enjoy the problems!

1. Find the prime factorization of $8^{11}9^{15}$.

2. In many ways can the letters in the word MACAROON be arranged?

3. Solve the equation
$$x + \frac{x}{3} + \frac{x}{5} = 0$$

4. Sam bought 5 pens and 4 notebooks at the store. If each pen cost 84 cents and each notebook cost \$3.17, how much change did Sam get if he paid with a twenty dollar bill?

5. Find one ordered pair of positive integers (m, n) which satisfies the equation $m^2 - 5n^2 = 1$.

6. If Hezy drives for three hours at 30 miles per hour followed by five hours at 46 miles per hour, what is his average speed in miles per hour for the whole trip?

7. Sandra reads a ten-volume history of Texas. Volume One is 110 pages long. Each subsequent volume is ten pages longer than the previous volume. She reads 50 pages per day except when reaching the end of a volume she stops reading for that day. How many days does it take Sandra to finish reading all ten volumes of this history of Texas?

8. The sum of two integers is 61. What is the largest possible value of their product?

9. A group of students plan to contribute equally to a charity. They have decided in advance the total amount of their contribution. If three students drop out of this plan, the remaining students would have to each contribute two dollars more to preserve the original gift. If six of the original students were to drop out of this plan, the remaining students would each have to contribute five dollars more to preserve the original gift. How many students are in the original group?

10. Simplify
$$(x + y - z)(x + y + z)$$

11. The graph of the equation $9x^2 - 16y^2 = 0$ is the union of two lines. What is the product of the slopes of those lines?

12. Four standard cubical fair dice are rolled. The probability of getting two distinct pairs (such as two 3's and two 5's) can be expressed in the form m/n, where m and n are relatively prime positive integers. Find the value of $m + n$.

13. What is the number of subsets of $\{1, 2, 3, 4, \cdots, 99, 100\}$ with an even number of even numbers? Recall that 0 is an even number. Answer in the form 2^n where n is an integer.

14. Find the number of ways to color the edges of a square if four colors are available and two colorings are considered the same if one can be rotated into the other.

15. Let a and b solve the system
$$\begin{cases} a + \dfrac{1}{b} = 7 \\ b + \dfrac{1}{a} = 9/14 \end{cases}$$
Find the sum of all possible values of ab.