Fall 2011 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. Two sweaters, a pair of wool socks, and a coat cost \$180. One sweater and the coat cost \$ 130. How much does one sweater and a pair of wool socks cost?

   (A) \$30 **(B)** \$40 **(C)** \$50 **(D)** \$60 **(E)** \$70

2. The expression

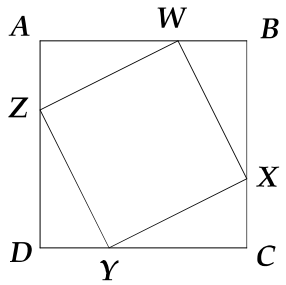
$$3\big((a+3b)4+2(5b+a)\big)$$

   is equivalent to the expression

   (A) $18a+22b$ **(B)** $15a+22b$ **(C)** $42a+42b$
   (D) $15a+66b$ **(E)** $18a+66b$

3. In square $ABCD$, square $WXYZ$ is inscribed in such a way that W is two-thirds of the way from A to B, X is two-thirds of the way from B to C, Y is two-thirds of the way from C to D, and Z is two-thirds of the way from D to A. If the area of $WXYZ$ is 100, what is the area of $ABCD$?

   (A) 150 **(B)** 160 **(C)** 170 **(D)** 180 **(E)** 200



4. From a regular deck of 52 cards three cards are dealt to you. What is the probability all three are red cards? Recall the red suites are hearts and diamonds.

   (A) $2/17$ **(B)** $1/8$ **(C)** $2/15$ **(D)** $1/7$ **(E)** $2/13$

5. If $f(3x+1) = \dfrac{2}{x+4}$, then $f(x+3) =$

(A) $\dfrac{6}{x+14}$ **(B)** $\dfrac{6}{3x+14}$ **(C)** $\dfrac{6}{x+11}$ **(D)** $\dfrac{6}{x+17}$ **(E)** $\dfrac{2}{x+14}$

6. A given cone's dimensions are modified as described in the responses below. Which response does **not** change the volume?

(A) double the height and halve the radius
(B) halve the height and double the radius
(C) quadruple the height and halve the radius
(D) halve the height and quadruple the radius
(E) quadruple the height and halve the radius twice

7. In how many ways can 10 be written as a sum of one or more positive integers if order does not matter and no integer can be repeated in a given sum? Thus, for instance, $4+6$ is considered the same as $6+4$, and $5+5$ is not allowed.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

8. Find the area enclosed by the graph of

$$|2y-1| + |2y+1| + 2|x| = 4$$

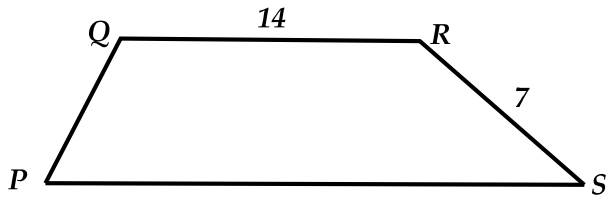
(A) 2 **(B)** 2.5 **(C)** 3 **(D)** 3.5 **(E)** 4

9. An ordered pair (m, n) of positive integers is called a *three-pair* if the interior angle of a regular polygon with m sides is three times the exterior angle of a regular polygon with n sides. How many *three-pairs* exist?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** more than 6

10. In trapezoid $PQRS$ as shown with $PS \parallel QR$, $QR = 14$, $RS = 7$, and $\angle R = 2\angle P$, the length of PS is

(A) 15 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 27



11. If p people consume m pounds of mashed potato in h hours, then the pounds of mashed potato consumed by m people in p hours equals:

(A) mph **(B)** $\frac{m}{ph}$ **(C)** $\frac{m^2}{ph}$ **(D)** $\frac{m^2}{h}$ **(E)** $\frac{p^2}{m}$

12. Find the sum

$$1 \cdot 25 + 2 \cdot 24 + 3 \cdot 23 + 4 \cdot 22 + \cdots + 24 \cdot 2 + 25 \cdot 1$$

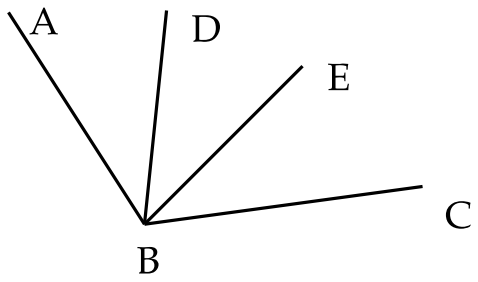
(A) 2500 **(B)** 2725 **(C)** 2800 **(D)** 2825 **(E)** 2925

13. How many of the numbers in this set below are irrational?

$$\{\sqrt{1.00}, \sqrt{1.01}, \sqrt{1.02}, \sqrt{1.03}, \cdots, \sqrt{3.98}, \sqrt{3.99}\}$$

(A) 299 **(B)** 294 **(C)** 290 **(D)** 286 **(E)** 150

14. $\angle ABC$ is trisected by rays $\overrightarrow{BD}$ and $\overrightarrow{BE}$ as shown. If the degree measure of $\angle DBC$ equals $3x - 5$ and that of $\angle ABC$ equals $5x - 22$, find the value of x.



(A) 17 **(B)** 19 **(C)** 23 **(D)** 29 **(E)** 31

15. How many factors of $51^5 \cdot 71^7 \cdot 91^9$ are perfect squares?

(A) 1 **(B)** 60 **(C)** 180 **(D)** 192 **(E)** 900

16. In acute triangle ABC, the intersection of its three altitudes, called the *orthocenter*, is labeled P. Given that $AP = 6$, $BP = 4$, and $BC = 10$, find AC.

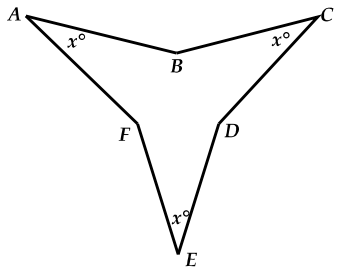
(A) $\sqrt{120}$ **(B)** $\sqrt{130}$ **(C)** $\sqrt{140}$ **(D)** $\sqrt{150}$ **(E)** $\sqrt{160}$

17. If $a = \dfrac{1110}{1111}$, $b = \dfrac{2221}{2223}$, and $c = \dfrac{3331}{3334}$ which of the following is true?

(A) $a > b > c$ **(B)** $b > a > c$ **(C)** $c > a > b$
(D) $c > b > a$ **(E)** $b > c > a$

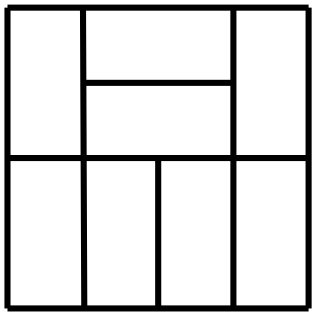
18. In non-convex hexagon $ABCDEF$, $AB = BC = CD = DE = EF = FA$ and $\angle A \cong \angle C \cong \angle E$. If the degree measure common to these three angles is x, what is the degree measure of $\angle ABC$ in terms of x?

(A) $120 + x$ **(B)** $180 - x$ **(C)** $90 + 2x$ **(D)** $180 - 3x$ **(E)** $60 + 3x$



19. In how many ways can a 4×4 nailed down board be tiled by eight 1×2 dominoes? One way to tile the board is shown below.

(A) 16 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 49



20. Let L_1 and L_2 be two intersecting lines. Let P be an arbitrary point of the plane determined by L_1 and L_2. Consider the following sequence of transformations in this plane. First, the point P is reflected across line L_1 to point Q. Second, point Q is reflected across line L_2 to point R. This sequence of transformations that maps point P to point R is equivalent to

(A) a translation
(B) a reflection about some third line
(C) a rotation about the point of intersection of the lines by an angle equal to the smaller angle formed by the lines
(D) a rotation about the point of intersection of the lines by an angle equal to twice the smaller angle formed by the lines
(E) a translation followed by a reflection about some third line