SPRING 2013 McNABB GDCTM CONTEST
GEOMETRY

NO Calculators Allowed

1. Express the fraction

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5}}}$$

in lowest terms.

(A) 1/8   (B) 11/15   (C) 15/21   (D) 16/21   (E) 21/16

2. The square root of 20000 lies between

(A) 130 and 131   (B) 140 and 141   (C) 141 and 142
(D) 142 and 143   (E) 10,000 and 10,001

3. How many positive factors does 2013 have?

(A) 6   (B) 8   (C) 10   (D) 12   (E) 14

4. Four indistinguishable roses and two indistinguishable tulips are to be arranged in a circle. Two such arrangements are considered to be the same if and only if each can be rotated into the other. How many distinct arrangements are possible?

(A) 3   (B) 4   (C) 8   (D) 12   (E) 15

5. When a cyclist gets a puncture she has just completed three-fourths of her route. She finishes her route by walking. If she spent twice as much time walking as biking, how many times faster does she bike than walk?

(A) 4   (B) 4.5   (C) 5   (D) 5.5   (E) 6

6. Which of the following equations has exactly two solutions over the real numbers?

(A) $x^2 - 6x + 9 = 0$ (B) $5x = 2(5 - 7x)$ (C) $|x + 8| = -5$
(D) $|x| = 12$ (E) $x^2 + 1 = 0$

7. If the equations $x^2 + ax + 21 = 0$ and $2x^2 + 19x + 35 = 0$ have a solution in common, what could be the value of the constant a?

   (A) -10   **(B)** -4   **(C)** -2   **(D)** 4   **(E)** 10

8. An off-center balance does balance when pan A has a weight of 600 grams while pan B has a weight of 900 grams. If a weight of 400 grams is added to pan A, how many grams must be added to pan B to restore the balance? Neglect the mass of the pans, beams, etc...

   (A) 400   **(B)** 500   **(C)** 600   **(D)** 700   **(E)** 900

9. Twenty-seven unit cubes are assembled to form a $3 \times 3 \times 3$ cube. If two of the unit cubes are then chosen at random, what is the probability they share a face?

   (A) 2/13   **(B)** 3/11   **(C)** 1/4   **(D)** 3/16   **(E)** 1/3

10. In pentagon $ABCDE$, $AB = AE = 3$, $BC = DE = 1$, $CD = 3$, $\angle B = \angle E$, and $\angle A$ is right. The area of this pentagon lies between

    (A) 6 and 7   **(B)** 7 and 8   **(C)** 8 and 9
    (D) 9 and 10   **(E)** 10 and 11

11. A rectangle with area 125 has its sides in the ratio of $4 : 5$. What is the perimeter of this rectangle?

    (A) 18   **(B)** 22.5   **(C)** 36   **(D)** 45   **(E)** 54

12. A solid opaque cube of side length 5 meters rests on flat ground. It is illuminated only by a powerful point-source of light located 5 meters above one of the cube's top corners. Find the area in square meters of the shadow cast by the cube on the ground.

    (A) 75   **(B)** 85   **(C)** $50\sqrt{2}$ **(D)** 91   **(E)** 100

13. For what value of the constant a do the three lines

$$2x + 5y = -7 \qquad 3x - 2y = 18 \qquad ax + 6y = 2$$

   all intersect at the same point?

    (A) 5   **(B)** 6   **(C)** 7   **(D)** 8   **(E)** 9

14. Two sides of a parallelogram lie along the lines $x - y + 1 = 0$ and $2x + 3y - 6 = 0$. If the diagonals of the parallelogram meet at the point $(1, 1/2)$, find the area of this parallelogram.

    (A) $3/2$ **(B)** 2 **(C)** $5/2$ **(D)** $18/7$ **(E)** 3

15. The first three figures of a certain sequence of figures are shown below on an equilateral triangle grid. Each successor figure is obtained recursively from its predeccesor by this rule: any two or more consecutive dots on a grid line generate new neighboring dots on that grid line, on either side, where no dot was before. All previous dots remain. How many dots does the 5th figure in this sequence have?

    (A) 34   **(B)** 36   **(C)** 57   **(D)** 59   **(E)** 64
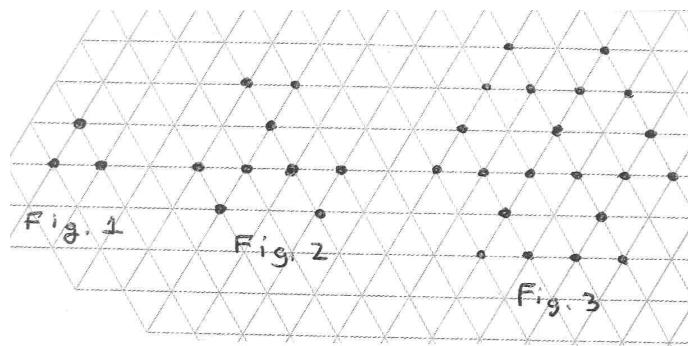
16. Given the three points $(2013, -1863)$, $(1776, -1812)$, and $(1181, -1492)$ in the coordinate plane, a fourth point (a, b) is called a *complementing* point if it along with the given three points form the vertices of a parallelogram. Find the sum of all the coordinates of all the complementing points of the given three points.

    (A) -197   **(B)** 0   **(C)** 216   **(D)** 631   **(E)** 783

17. Quadrilateral $PQRS$ is inscribed in a circle. Segments PQ and SR are extended to meet at T. If $\angle SPQ = 80°$ and $\angle PQR = 130°$, find in degrees the measure of $\angle T$.

    (A) 50   **(B)** 53   **(C)** 57   **(D)** 60   **(E)** 61

18. A circle of radius 9 is externally tangent to a second circle of radius b. If a common tangent to the two circles has length 12, what is the value of b?

    (A) 3.5   **(B)** 4   **(C)** 6   **(D)** 7.5   **(E)** 9