Spring 2014 GDCTM/McNabb Geometry Contest

NO Calculators Allowed

1. How many times do the graphs of the functions $f(x) = 1000|x|$ and $g(x) = x^2/1000$ intersect?

2. Find all value(s) of the constant a so that the line $ax + 2y = 7$ is perpendicular to the line $ax - 3y = 2$.

3. Four cards are drawn randomly from a standard 52 card deck. What is the probability that no two of these cards belong to the same suit?

4. A running back tries to first elude tackler A and then tackler B. His probability of eluding A is $2/3$ and his probability of eluding both A and B is $1/2$. What is the probability that the running back eludes B once he has eluded A?

5. Find the sum of the square roots of the roots of the quadratic equation $x^2 - 39x + 25 = 0$.

6. Solve the system

$$\begin{aligned} ab^2c^2 &= -18 \\ a^3bc^2 &= 12 \\ ab^2c &= -36 \end{aligned}$$

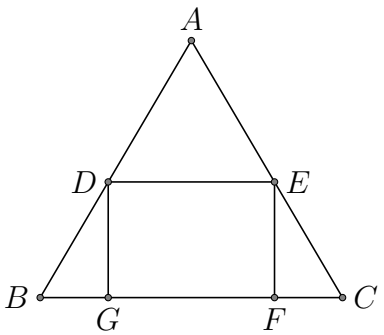
7. The area of a trapezoid is 24. If all of the sides of the trapezoid are doubled while keeping all the corresponding angles the same, what is the area of the new trapezoid?

8. Find the area of pentagon $ABCDE$ if the coordinates of the vertices are: $A = (0, 0), B = (1, -4), C = (9, 2), D = (0, 5), E = (-4, 2)$.

9. How many pairs of vertical angles are formed by 7 distinct lines all passing through the same point?

10. Eight cows graze a pristine field bare in 40 days. It would take 15 cows just 12 days to graze the same pristine field bare. How many days would it take 10 cows to graze that same pristine field bare? Assume that the grass in this field grows at a constant rate and the cows graze at a constant rate.

11. Suppose square $DEFG$ is inscribed in equilateral triangle ABC as shown. Find the ratio of the area of ABC to the area of $DEFG$.



12. Nine colorless unit squares are assembled into a 3 by 3 square. Two squares are to be colored red and two others yellow. In how many different ways can this be done if two colorings are considered to be the same if one can be rotated to coincide with the other?

13. Point P can be any point on a fixed circle of radius seven, while point Q can be any point in the plane of the circle at a distance of three from point P. Find the area of the set of all possible locations of the point Q.

14. Recall that a *Pythagorean Triple* has the form (a, b, c) where a, b, and c are positive integers satisfying $a^2 + b^2 = c^2$. Find a Pythagorean Triple in which a or b equals 17.

15. In a square of side length six, segments are drawn from the midpoint of each side to the opposite vertices of the square, forming a convex octagon as shown below. Find the area of this octagon.