# Fall 2016 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. In parallelogram $ABCD$, diagonals AC and BD intersect at point E. If $\angle BAC = 70°$ and $\angle BDC = 40°$, find $\angle AEB$.

2. In parallelogram $ABCD$, the angle bisectors of $\angle A$ and $\angle B$ meet at point E, which happens to lie on segment CD. Suppose that $AB = 20$ and that $\angle AEB$ is a right angle. Find AD.

3. A rectangle has sides 4 and 6. The angle bisectors of each of the interior angles of the rectangle are drawn, forming a quadrilateral inside the rectangle. Find the area of this quadrilateral.
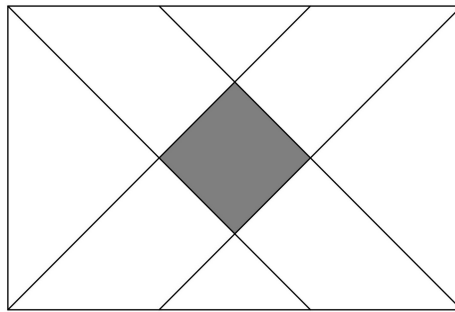
4. On a number line, the number $4/5$ ths of the way from a to $8/9$ is $7/9$. Find $3a$.

5. A convex polygon has exactly 135 diagonals. How many edges does it have?

6. In the figure, each arc is a quarter circle, for circles of radius 1, 2, and 3. What is the area of the shaded region enclosed by these arcs?
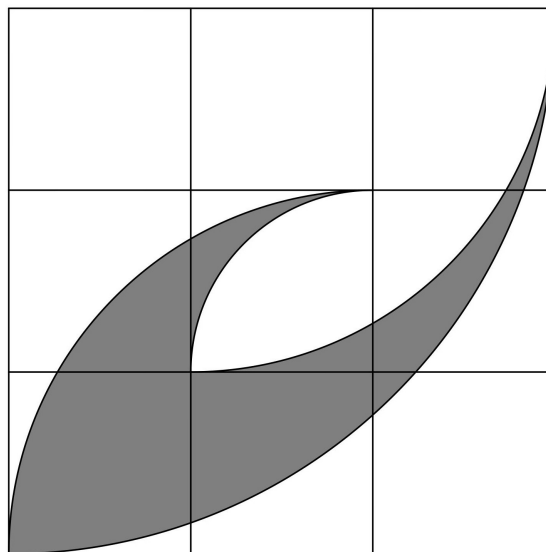
7. At each lattice point on the coordinate plane you can either step one unit up, one unit down, one unit left, or one unit right. In how many different ways can you start at the origin and in six such steps arrive at the point $(2, 2)$. You can revisit lattice points, including your final destination $(2, 2)$. Note: a point is a lattice point if both of its coordinates are integers.

8. Suppose that T is a regular tetrahedron of side length 1. An ant begins at the midpoint of an edge and wants to travel to the midpoint of the opposite edge. If the ant travels along the surface of the tetrahedron, what is the shortest possible distance in which the ant could make the trip?

9. Find the radius of the circumcircle of the triangle with side lengths 13, 14, and 15.

10. A trapezoid has its parallel bases of lengths 7 and 1. Find the length of the segment that is parallel to the bases and divides the trapezoid into two smaller trapezoids with equal areas.

11. Three congruent circles of radius 10 are mutually externally tangent. Find the area of the curved region in between the three circles.

12. An equilateral triangle ABC with side length 10 is inscribed in a circle. Diameter MN is drawn, parallel to BC and intersecting AB and AC at points P and Q respectively. Find QC.

13. A frustum has a larger radius of 5, a smaller radius of 2, and a height of 6. An inverted cone with base the same as the smaller base of the frustum and apex on the larger base of the frustum is removed from the frustum. Find the volume of the frustum that remains.

14. Eight points lie evenly spaced on a circle. Find the number of ways to draw four non-intersecting chords that join these points. Note that each of the eight points will be an endpoint of exactly one of these chords.

15. What is the maximum number of lattice points that can lie strictly inside an equilateral triangle of side length 7?