# Fall 2010 McNabb GDCTM Contest
# Algebra II

## NO Calculators Allowed

1. An automobile goes $y/9$ yards in $d$ seconds. How many feet does it travel in two minutes time?

   **(A)** $\frac{40y}{d}$ **(B)** $\frac{40d}{3y}$ **(C)** $\frac{3y}{40d}$ **(D)** $120yd$ **(E)** $\frac{120y}{d}$

2. The well-known formula $f = (9/5)c + 32$ relates the temperature $f$ in Fahrenheit to the temperature $c$ in Celcius. For how many values of $f$ satisfying $32 \leq f \leq 212$, will the temperature be an integer in both of these scales?

   **(A)** 9 **(B)** 10 **(C)** 19 **(D)** 20 **(E)** 21

3. If $f(\sqrt{x}) = \frac{x+3}{13}$ and $g(x^2) = x^4 + 3x^2 - 22$, then find $f(g(4))$.

   **(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

4. A square is inscribed in a right triangle with sides of length 3, 4, and 5, so that one of the sides of the square is contained in the hypotenuse of the right triangle. What is the side length of the square?

   **(A)** $\frac{60}{37}$ **(B)** 2 **(C)** $\frac{12}{5}$ **(D)** 3 **(E)** cannot be determined

5. The arithmetic mean of $a$, $b$, and $c$ is 7 and the arithmetic mean of $a^2$, $b^2$, and $c^2$ is 55. What is the arithmetic mean of $ab$, $bc$, and $ac$?

   **(A)** 24 **(B)** 31 **(C)** 46 **(D)** 48 **(E)** 92

6. In how many ways can the the letters in the string $ABECEDA$ be arranged so that the consonants are in alphabetical order?

   **(A)** 90 **(B)** 105 **(C)** 120 **(D)** 180 **(E)** 210

7. The graph of the quadratic function $f(x) = ax^2 + bx + c$ contains the points $(-1, 6)$, $(7, 6)$, and $(1, -6)$. What is the minimum value of $f(x)$?

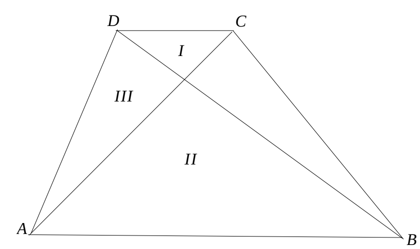
**(A)** -36 **(B)** -26 **(C)** -20 **(D)** -10 **(E)** -6

8. Let $a$, $b$, and $c$ be positive real numbers. Supposing that $ab = kc$, $ac = lb$, and $bc = ma$, then $c$ must equal

**(A)** $lm$ **(B)** $\sqrt{klm}$ **(C)** $\sqrt{\frac{l}{m}}$ **(D)** $k\sqrt{lm}$ **(E)** $\sqrt{lm}$

9. The real number $\sqrt{16 + \sqrt{220}}$ can be expressed in the form $\sqrt{A} + \sqrt{B}$, where $A$ and $B$ are integers and $A > B$. What is the value of $A - B$?

**(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10




10. In trapezoid $ABCD$, the area of region I is 9 and the area of region II is 16. What is the area of region III?

**(A)** 10 **(B)** 11 **(C)** 12 **(D)** 12.5 **(E)** cannot be determined

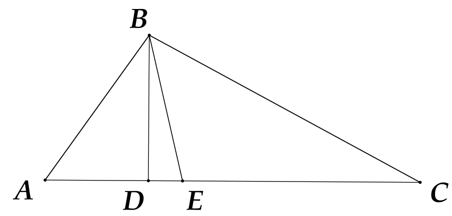
11. The polynomial

$$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

can be factored as $P_2(x) \cdot P_6(x)$ where $P_2$ and $P_6$ are polynomials with integer coefficients and have degrees 2 and 6 respectively. Find the sum of the coefficients of $P_2$.

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

12. In $\triangle ABC$, $\angle A = 60^\circ$, $C = 40^\circ$, $BD \perp AC$, and $\overrightarrow{BE}$ bisects $\angle ABC$. Find the measure of $\angle DBE$ in degrees.




**(A)** 8 **(B)** 10 **(C)** 12 **(D)** 14 **(E)** 20

13. A bag contains 4 quarters and 2 dimes. If 3 coins are randomly removed from the bag, what is the expected total value in cents of these three coins?

**(A)** 50 **(B)** 55 **(C)** 60 **(D)** 65 **(E)** 75

14. The set $S$ contains seven numbers whose mean is 202. The mean of the four smallest numbers in S equals 100, while the mean of the four largest numbers in $S$ equals 300. What is the median of all the numbers in $S$?

**(A)** 184 **(B)** 186 **(C)** 192 **(D)** 196 **(E)** 200

15. Let $a$ and $b$ be positive constants. If $x$ is a solution of

$$\sqrt{x+a} + \sqrt{x+b} = \sqrt{a+b}$$

then $x$ must equal

**(A)** 0 **(B)** $\frac{a+b}{2}$ **(C)** $-\frac{ab}{a+b}$
**(D)** $-\left(\frac{1}{a} + \frac{1}{b}\right)$ **(E)** $-\frac{2}{a+b}$

16. While Xerxes marched on Greece his army streched out for 50 miles. A dispatch rider had to ride from the rear to the head of the army, then instantly turn about and return to the rear. While he did this, the army advanced 50 miles. How many miles did the rider ride?

**(A)** 100 **(B)** $50 + 50\sqrt{2}$ **(C)** $100\sqrt{2}$
**(D)** 150 **(E)** $50 + 100\sqrt{2}$

17. The sum of two of the roots of $p(x) = 4x^3 + 8x^2 - 9x - k$, where $k$ is a constant, is zero. Find the value of $k$.

**(A)** 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 200

18. Let $f$ be a function such that $f(x + y) = f(xy)$ for all real numbers $x$ and $y$. If it is also known that $f(5) = 5$, determine the value of $f(25)$.

**(A)** 1 **(B)** 5 **(C)** 10 **(D)** 20 **(E)** 25

19. Hezy and Zeke were employed at different daily wages. At the end of a certain number of days Hezy received $300, while Zeke, who had been absent from work two of those days, received only $192. However, had it been the other way around, had Zeke worked all those days and Hezy been absent twice, then both would have received the same amount. What was Hezy's daily wage?

**(A)** 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

20. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

**(A)** 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16