

FALL 2011 McNABB GDCTM CONTEST
ALGEBRA II

NO Calculators Allowed

1. A certain number is doubled. The result is then increased by nine. This result is decreased by 3. If this last number is 28, what was the original number?

(A) -4 (B) 0 (C) 7 (D) 11 (E) 28

2. The expression

$$3((a + 3b)4 + 2(5b + a))$$

is equivalent to the expression

(A) $18a + 22b$ (B) $15a + 22b$ (C) $42a + 42b$
(D) $15a + 66b$ (E) $18a + 66b$

3. How many of the numbers in this set below are irrational?

$$\sqrt{1.00}, \sqrt{1.01}, \sqrt{1.02}, \sqrt{1.03}, \dots, \sqrt{3.98}, \sqrt{3.99}$$

(A) 299 (B) 294 (C) 290 (D) 286 (E) 150

4. If $f(3x + 1) = \frac{2}{x + 4}$, then $f(x + 3) =$

(A) $\frac{6}{x + 14}$ (B) $\frac{6}{3x + 14}$ (C) $\frac{6}{x + 11}$ (D) $\frac{6}{x + 17}$ (E) $\frac{2}{x + 14}$

5. An ordered pair (m, n) of positive integers is called a *three-pair* if the interior angle of a regular polygon with m sides is three times the exterior angle of a regular polygon with n sides. How many *three-pairs* exist?

(A) 0 (B) 2 (C) 4 (D) 6 (E) more than 6

6. Amanda, Brice, and Carl all start working at SellMore with the same salary on the same day. They receive the following percent raises in their salary at the end of each of the first two years in order:

Amanda: 5%, 3%

Brice: 3%, 5%

Carl: 4%, 4%

Which of them earns the most total over their first three years at SellMore?

- (A) Amanda (B) Brice (C) Carl
(D) Amanda and Brice tied for the most (E) All three tied for the most

7. Let a , b , c , and d be non-zero constants. If the lines $ax + by = 0$ and $cx + dy = 0$ are perpendicular then which of these quantities must be zero?

- (A) $ad - bc$ (B) $ac + bd$ (C) $ac - bd$
(D) $ad + bc$ (E) $a^2 + b^2 + c^2 + d^2$

8. If p people consume m pounds of mashed potato in h hours, then the pounds of mashed potato consumed by m people in p hours equals:

- (A) mph (B) $\frac{m}{ph}$ (C) $\frac{m^2}{ph}$ (D) $\frac{m^2}{h}$ (E) $\frac{p^2}{m}$

9. If $a < b < c < d < e$ which of the following must be true?

- (A) $ab < cd$
(B) $c - a < e - c$
(C) $a^2 < e^2$
(D) $ad + bc < ac + bd$
(E) $b + d < 2c$

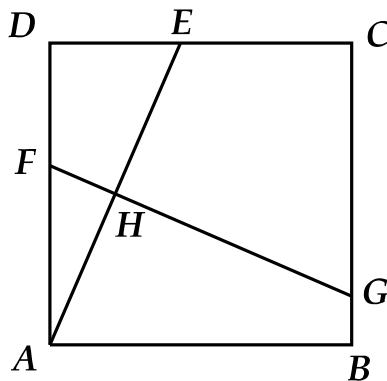
10. Find the coefficient of x^{30} in the expansion of

$$(1 + x^3)(1 + x^6)(1 + x^9)(1 + x^{12}) \cdots (1 + x^{27})(1 + x^{30})$$

- (A) 9 (B) 10 (C) 33 (D) 43 (E) 57

11. In how many ways can the letters in the word *monsoon* be arranged so that the second *n* occurs before the third *o*?
- (A) 210 (B) 216 (C) 252 (D) 256 (E) 260
12. Suppose $a + b^{-1} = 4$ and $b + a^{-1} = 4/3$. If $a > b^{-1}$ what is the value of ab ?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
13. In $\triangle ABC$, points E and F lie on \overline{BC} and \overline{AC} respectively. Let \overline{AE} and \overline{BF} intersect at G . If $\frac{AF}{FC} = \frac{3}{5}$ and G is the midpoint of \overline{BF} , then find the ratio $\frac{CE}{EB}$.
- (A) 3 (B) $7/2$ (C) $8/3$ (D) $3/2$ (E) $11/4$
14. Independently of each other, Hezy and Zeke each randomly pick a real number between 0 and 3. What is the probability that their choices differ by at least 1?
- (A) $1/9$ (B) $1/3$ (C) $1/2$ (D) $4/9$ (E) $5/9$
15. Find the area enclosed by the graph of
- $$|2y - 1| + |2y + 1| + 2|x| = 4$$
- (A) 2 (B) 2.5 (C) 3 (D) 3.5 (E) 4
16. Find the sum
- $$1 \cdot 25 + 2 \cdot 24 + 3 \cdot 23 + 4 \cdot 22 + \cdots + 24 \cdot 2 + 25 \cdot 1$$
- (A) 2500 (B) 2725 (C) 2800 (D) 2825 (E) 2925
17. Square $ABCD$ of side-length 2 is inscribed in a circle. If chord AP bisects segment BC of the square, what is the square of the length of chord AP ?
- (A) 7 (B) 7.2 (C) 7.4 (D) 7.5 (E) 7.7

18. Let $f(x) = ax^2 + bx + c$, where a , b , and c are all non-zero constants. If $c = \frac{b^2}{4a}$, then the graph of f must
- (A) be symmetric with respect to the y axis
 - (B) be symmetric with respect to the x axis
 - (C) be tangent to the x axis
 - (D) be tangent to the y axis
 - (E) have a maximum point
19. Inside square $ABCD$ point E lies on side \overline{CD} with $\frac{CE}{ED} = \frac{5}{3}$. The perpendicular bisector of \overline{AE} intersects the square at points F and G and intersects \overline{AE} at H , as shown. Find the ratio $\frac{FH}{HG}$
- (A) $\frac{3}{13}$ (B) $\frac{1}{4}$ (C) $\frac{2}{7}$ (D) $\frac{1}{3}$ (E) $\frac{4}{11}$



20. In acute triangle ABC , the intersection of its three altitudes, called the *orthocenter*, is labeled P . Given that $AP = 6$, $BP = 4$, and $BC = 10$, find AC .
- (A) $\sqrt{120}$ (B) $\sqrt{130}$ (C) $\sqrt{140}$ (D) $\sqrt{150}$ (E) $\sqrt{160}$