

FALL 2013 McNABB GDCTM CONTEST  
ALGEBRA TWO

NO Calculators Allowed

1. If  $x$  ounces of cleaner clean  $y$  square feet of floor, how many square yards of floor can be cleaned by  $y$  gallons of this cleaner?
2. A set  $S$  of ordered pairs is said to be *transitive* if whenever  $(a, b)$  and  $(b, c)$  belong to  $S$  then so does  $(a, c)$ . Is this set  $S$  below transitive?

$$S = \{(6, 13), (4, 8), (5, 7), (6, 10), (3, 5), (10, 13), (3, 7), (1, 5), \\ (3, 10), (1, 4), (1, 7), (9, 6)\}$$

Answer Yes or No.

3. Find the coefficient of  $x^2$  when  $(1 + x + x^2)^6$  is expanded and simplified.
4. Find the area of the region  $T = \{(x, y) : |x| + 3|y| \leq 4\}$ .
5. Two congruent circles (in the same plane) do not intersect. Their centers are a distance 10 units apart. The length of their common internal tangent is 8 units. What is the radius of this pair of congruent circles?
6. Segment  $AD$  is an altitude of equilateral triangle  $ABC$  and segment  $DE$  is an altitude of triangle  $CDA$ . Find the ratio  $AE/EC$ .
7. Let  $f(x, y) = yx^2 - (2y + 1)x + y$ . Solve  $f(x, 6) = 0$ .
8. If the point  $(x, y)$  satisfies

$$x^3 - 71x = y^3 - 71y$$

but does not satisfy  $x = y$  then what is the value of  $x^2 + xy + y^2$ ?

9. Find the set of all values of the parameter  $a$  so that the graph of the parabola  $y = ax^2 + 2x + 4a$  never enters the third quadrant  $III$ . Recall that  $III = \{(x, y) : x < 0 \text{ and } y < 0\}$ .
10. Find the minimum possible value of the expression

$$(x - 9)^2 + (x - 7)^2 + (x - 5)^2 + (x + 5)^2 + (x + 7)^2 + (x + 9)^2$$

11. Find the value of the index  $n$  if

$$\sqrt[3]{r^n \sqrt{\left(\frac{1}{r}\right)}} \sqrt[4]{r} = r^{\frac{7}{24}}$$

12. In  $\triangle ABC$  with point  $D$  on segment  $AC$  so that  $AD : DC = 2 : 5$ , draw segment  $BD$  with points  $E$  and  $F$  on  $BD$  in order  $B - E - F - D$ . If the area of  $\triangle EFC$  is 5 and the area of  $\triangle ABC$  is 30, find the area of  $\triangle AEF$ .
13. Let  $m$  and  $n$  be two relatively prime positive integers. Find the maximum possible value of the gcd of  $m + 24n$  and  $n + 24m$ .
14. In which quadrant (I, II, III, or IV) of the complex plane does  $(-\sqrt{3} + 3i)^{17}$  lie?
15. The point  $(5, -4)$  lies on the graph of  $y = -2f(3(x + 6)) - 8$ . What corresponding point lies on the graph of  $y = (1/2)f(x - 2) + 5$ ?