

SPRING 2018 McNABB GDCTM CONTEST
ALGEBRA TWO

NO Calculators Allowed/ 60 Minutes

1. The positive integers are put in a rectangular grid in the following way

1	2	3	4	5	6	7	8	9
18	17	16	15	14	13	12	11	10
19	20	21	22	23	24	25	26	27
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

What is the number at the top of the column which contains the number 70?

2. A solution of 18 liters is 20% acid. How many liters of 68% acid must be added to the original solution to form a solution that is 36% acid?
3. Find the maximum possible value of $a + b + c$ if a , b , and c must satisfy

$$\begin{aligned} 4a + b + 9c &\leq 13 \\ 2a + 18b + 7c &\leq 22 \\ 21a + 8b + 11c &\leq -15 \end{aligned}$$

4. A train passes a standing observer in seven seconds and a 378 yard platform station in 25 seconds. How long in yards is the train?
5. Let a be a fixed positive real number. Find the area of the triangle formed by the three lines

$$\begin{aligned} y &= ax \\ y &= \frac{x}{a} \\ x + y &= a \end{aligned}$$

in terms of a .

6. In trapezoid $ABCD$ with $AB \parallel CD$ and $AB/CD = 1/6$, draw diagonals AC and BD intersecting at point E . Find the ratio of the area of $ABCD$ to the area of ABE .
7. How many integers n satisfy

$$\frac{15n}{13} - 8 < \frac{11n}{9} < \frac{13n}{11} + 8$$

8. One counter is in a pouch. It is with equal probability either black or white. A white counter is added to the pouch. Next, one counter is randomly drawn from the pouch and it turns out to be white. What is the probability that the remaining counter in the pouch is black?
9. Find the period of the function

$$f(x) = \cos(7x/3) + \sin(3x/7)$$

where x is measured in radians.

10. Find the sum

$$i + i^2 + i^3 + i^4 + i^5 + i^6 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13} + i^{15} + \dots + i^{139} + i^{141}$$

where all the powers of i that have exponents that are multiples of seven have been omitted. Here i is a square root of -1 .

11. Factor $x^4 - 6x^3 + 9x^2 - 4$ into two quadratic polynomials with integer coefficients.

12. Let r and s be the roots of the monic quadratic $x^2 + 3x - 7$. If the monic quadratic $x^2 + bx + c$ has roots $r + 2$ and $s + 2$, find the value of bc .

13. A peculiar cat climbs stairs by taking steps only two or three at a time. Bored one day, the curious cat climbs over and over again the same stair, and finds that it can climb this set of stairs in exactly 114 different ways. How many steps does the staircase have?

14. Find the remainder when the polynomial

$$x^{30} + x^{24} + x^{18} + x^{12} + x^6 + 1$$

is divided by $x^5 + x^4 + x^3 + x^2 + x + 1$.

15. Integers a and b have the property that the cubic equations

$$x^3 + 10x^2 - 16x + a = 0$$

$$x^3 + 18x^2 + 88x + b = 0$$

share exactly two real roots. Find the value of $a + b$.