Spring 2019 McNabb GDCTM Contest
Algebra Two

NO Calculators Allowed/ 60 Minutes

Assume all variables are real unless otherwise stated in the problem.

1. Write 52 as the sum of three distinct primes. Only one way of doing this needs to be written down.

2. How many numbers are in the list

$$-21, -17, -13, \cdots, 499$$

   ?

3. Sam bought 5 pens and 4 notebooks at the store. If each pen cost 84 cents and each notebook cost $3.17, how much change did Sam get if he paid with a twenty dollar bill?

4. A group of students plan to contribute equally to a charity. They have decided in advance the total amount of their contribution. If three students drop out of this plan, the remaining students would have to each contribute two dollars more to preserve the original gift. If six of the original students were to drop out of this plan, the remaining students would each have to contribute five dollars more to preserve the original gift. How many students are in the original group?

5. What is the range of the function $f(x) = \ln(1 + 2^x)$? Answer in interval notation.

6. The angle bisectors of the angles of $\triangle ABC$ meet at point D. If $\angle A = 100°$, find the measure of $\angle BDC$ in degrees.

7. What is the period of the trigonometric function

$$f(x) = \sin 5x + \cos 4x + \tan 3x$$

   ? Note that x is assumed to be in radians.

8. Let a and b solve the system

$$\begin{cases} a + \dfrac{1}{b} = 7 \\ b + \dfrac{1}{a} = 9/14 \end{cases}$$

   Find the sum of all possible values of ab.

9. A piece of wire is wrapped in a spiral around a cylinder of radius $3/\pi$ and height 12 in such a way that it completes two revolutions as it goes from bottom to top of the cylinder. How long is the wire?

10. The sides of $\triangle ABC$ have lengths $AB = \sin 27°$, $BC = \cos 10°$, and $CA = \cos 17°$. Find the measure of $\angle A$ in degrees.

11. Solve

$$\sqrt{3+x} + \sqrt{3-x} > 3$$

Answer in interval notation.

12. When the binomial $\left(2a + \dfrac{1}{\sqrt{a}}\right)^9$ is expanded and simplified, what is the value of the constant term?

13. Two ladders, one of length 25 feet the other of length 17 feet criss-cross each other across an alley. The foot of each ladder is up against the base of the walls facing each other. The point at which the two ladders cross each other is $40/7$ feet above the alley. How wide is the alley?

14. Let z be a complex number. Find the solutions of

$$z^2 - (3-i)z + (8+i) = 0$$

Note that here $i = \sqrt{-1}$. Answers must be in standard complex form $a + bi$ where a and b are real numbers.

15. Solve

$$6(\log_8 x) \cdot (\log_2 x) + 6 \log_4 x = -1$$