Spring 2010 McNabb GDCTM Contest
Level J1

1. How many diagonals does a hexagon have?

   (**A**) 5   (**B**) 6   (**C**) 7   (**D**) 8   (**E**) 9

2. Amy is in a line of 40 people at the movies. Ahead of her stand 12 people. How many stand behind her?

   (**A**) 13   (**B**) 27   (**C**) 28   (**D**) 29   (**E**) 39

3. What is the smallest positive integer which has the five smallest primes as factors?

   (**A**) 209   (**B**) 210   (**C**) 2310   (**D**) 15015   (**E**) 100000

4. At the post office Amy spent a total of $ 5.00 to buy some 43 cent stamps and some 5 cent stamps. How many 5 cent stamps could she have bought?

   (**A**) 7   (**B**) 10   (**C**) 11   (**D**) 13   (**E**) 14

5. The average of two positive numbers is equal to twice the smaller of the two numbers. How many times greater is the larger number than the smaller?

   (**A**) 1.5 times   (**B**) 2 times   (**C**) 2.5 times   (**D**) 3 times   (**E**) 4 times

6. A kangaroo is 200 feet from a rabbit, when the kangaroo starts chasing the rabbit. Both immediately start hopping in the same direction. For each 13 foot leap of the kangaroo the rabbit takes two 4 foot leaps. From the time the chase began until the rabbit is caught, how many leaps did the rabbit take?

   (**A**) 20   (**B**) 40   (**C**) 80   (**D**) 100   (**E**) 200

7. When three distinct numbers from the set $\{9, 8, -2, -4, -5\}$ are multiplied, the largest possible product is

   (**A**) 64   (**B**) 90   (**C**) 160   (**D**) 180   (**E**) 360

8. The Elm school girls' basketball team has 11 girls on the team. The team will play 22 games this season, with each game lasting 32 minutes. The coach arranges for each girl to have the same total playing time by the end of the season. How many total minutes playing time would each girl end up with at the end of the season?

   (**A**) 64   (**B**) 128   (**C**) 200   (**D**) 320   (**E**) 440

9. Three pomegranates and one pineapple weigh as much as sixteen plums. Four plums and one pomegranate weight as much as one pineapple. How many pomegranates weigh as much as 3 pineapples?

   (**A**) 5   (**B**) 6   (**C**) 7   (**D**) 9   (**E**) 11

10. A box contains 10 red marbles, 11 blue marbles, and 12 green marbles. What is the fewest number of marbles you must pull out of the box to be sure of getting at least 5 of the same color?

(A) 5   (B) 10   (C) 13   (D) 26   (E) 28

11. What is the area of the quadrilateral in the coordinate plane with vertices whose coordinates are (in order): $(0,0)$, $(7,1)$, $(4,4)$, and $(2,11)$?

(A) 30   (B) 31   (C) 31.5   (D) 32   (E) 33

12. Ronald has an unlimited number of 5 cent and 7 cent stamps. What is the largest amount of postage (in cents) that he cannot make with these stamps?

(A) 16   (B) 22   (C) 23   (D) 79   (E) 99

13. How many elements are in the set $\{7, 11, 15, 19, \cdots, 403\}$?

(A) 99   (B) 100   (C) 101   (D) 102   (E) 397

14. Abigail, Brice, and Carl all start out with some one-dollar bills in their pockets. In particular, Carl starts with 4 one-dollar bills. Brice then gives half of his dollar bills to Abigail and the other half to Carl. Then Abigail gives half of her bills to Brice and the other half to Carl. Finally, Carl gives half of his bills to Abigail and keeps the rest. If now, at the end of these exchanges, Abigail, Brice, and Carl all have 8 one-dollar bills, how many such bills did Abigail have to begin with?

(A) 8   (B) 9   (C) 10   (D) 11   (E) 12

15. Farmer Ben sells thimbleberry jam in cylindrical jars. The company that supplies his jars is discontinuing the size jar he is currently using. Farmer Ben will have to order jars that have a diameter 10% less than his current jar. To maintain the volume of his current jar, the new jars he orders should have a height what percent greater than the current ones? Answer to the nearest percent.

(A) 11   (B) 23   (C) 24   (D) 25   (E) 124

16. Let a, b, c, and d be positive real numbers such that $abc = 12$, $bcd = 6$, $acd = 125$, and $abd = 3$. Then $abcd$ equals

(A) 12   (B) 15   (C) 30   (D) 45   (E) 60

17. In throwing four fair cubical dice, what is the probability of obtaining two distinct doubles?

(A) $\dfrac{5}{72}$ (B) $\dfrac{7}{36}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{5}{16}$ (E) $\dfrac{3}{8}$

18. If a is a multiple of 14 and b is a multiple of 21, then what is the largest integer that must be a factor of any integer of the form $9a + 8b$?

(**A**) 84   (**B**) 42   (**C**) 21   (**D**) 14   (**E**) 8

19. Each vertex of a cube is randomly colored red or blue with each color being equally likely. What is the probability that every pair of adjacent vertices have different colors?

(**A**) 0   (**B**) 1/128   (**C**) 1/64   (**D**) 1/32   (**E**) 1/2

20. The value of the expression

$$(1 - (2 - (3 - (4 - (5 - (\cdots - (n)))))\cdots)$$

for n a positive even integer is equal to

(**A**) $-n$ (**B**) $-n/2$ (**C**) 0   (**D**) $n/2$ (**E**) $n - 3$

21. A 4 inch by 4 inch square board is subdivided into sixteen 1 inch by 1 inch squares in the usual way. Four of the smaller squares are to be painted white, four black, four red, and four blue. In how many different ways can this be done if each row and each column of smaller squares must have one square of each color in it? (The board is nailed down: it can not be rotated or flipped).

(**A**) 576   (**B**) 864   (**C**) 1152   (**D**) 1200   (**E**) 1600

22. The sequence $1, 3, 6, 10, 15, 21, \cdots$ is called the sequence of *triangular* numbers, because those numbers of dots can be arranged to form equilateral triangles. For instance, the triangular number 10 occurs in the set-up of bowling pins. What is the smallest triangular number greater than 1000?

(**A**) 2^{10} (**B**) 1035   (**C**) 1021   (**D**) 1010   (**E**) 1006

23. In how many ways can five distinct books be arranged in a bookcase with 3 shelves, each shelf capable of holding all five books?

(**A**) 19   (**B**) 120   (**C**) 360   (**D**) 840   (**E**) 2520

24. The probability that Gerald wins any given game of HORSE is 3/5. Next Saturday, Gerald will play exactly five games of HORSE. What is the probability that he will win exactly three of them?

(**A**) $\dfrac{108}{3125}$ (**B**) $\dfrac{3}{5}$ (**C**) $\dfrac{216}{625}$ (**D**) $\dfrac{9}{25}$ (**E**) 1

25. Seven stacks are made each consisting of seven half-dollar coins. One entire stack is made of counterfeit coins. All other stacks have true half-dollars. You know the weight of true half-dollars in grams. And each counterfeit half-dollar weighs exactly one gram more than the true coin. You can weigh the coins or any subset of them on a digital scale (similar to a regular bathroom scale) which outputs in grams. What is the minimum number of weighings needed to determine which stack is the counterfeit one?

(**A**) 1   (**B**) 2   (**C**) 3   (**D**) 4   (**E**) 7