Fall 2010 McNabb GDCTM Contest
Pre-Calculus

NO Calculators Allowed

1. The well-known formula $f = (9/5)c + 32$ relates the temperature f in Fahrenheit to the temperature c in Celcius. For how many values of f satisfying $32 \leq f \leq 212$, will the temperature be an integer in both of these scales?

   (A) 9 **(B)** 10 **(C)** 19 **(D)** 20 **(E)** 21

2. A positive integer has the interesting property that when expressed as a three digit base-7 number, those digits are the reverse of its digits when expressed as a base-9 number. What is this number expressed in normal form as a base-10 number?

   (A) 124 **(B)** 241 **(C)** 248 **(D)** 428 **(E)** 503

3. What is the maximum number of times that the graph of a polynomial of degree six can intersect the graph of a polynomial of degree five?

   (A) 1 **(B)** 5 **(C)** 6 **(D)** 11 **(E)** 30

4. The arithmetic mean of a, b, and c is 7 and the arithmetic mean of a^2, b^2, and c^2 is 55. What is the arithmetic mean of ab, bc, and ac?

   (A) 24 **(B)** 31 **(C)** 46 **(D)** 48 **(E)** 92

5. Given the arithmetic series

   $$S = 3 + 6 + 9 + \cdots + 99$$
   $$T = 4 + 8 + 12 + \cdots + 132$$

   what is the value of the ratio $S : T$?

   (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** $\frac{5}{6}$

6. A square is inscribed in a right triangle with sides of length 3, 4, and 5, so that one of the sides of the square is contained in the hypotenuse of the right triangle. What is the side length of the square?

   (A) $\frac{60}{37}$ **(B)** 2 **(C)** $\frac{12}{5}$ **(D)** 3 **(E)** cannot be determined

7. In how many ways can the the letters in the string $ABECEDA$ be arranged so that the consonants are in alphabetical order?

   (A) 90 **(B)** 105 **(C)** 120 **(D)** 180 **(E)** 210

8. Hezy and Zeke were employed at different daily wages. At the end of a certain number of days Hezy received \$300, while Zeke, who had been absent from work two of those days, received only \$192. However, had it been the other way around, had Zeke worked all those days and Hezy been absent twice, then both would have received the same amount. What was Hezy's daily wage?

   (A) 30 **(B)** 40 **(C)** 50 **(D)** 60 **(E)** 70

9. The real number $\sqrt{16+\sqrt{220}}$ can be expressed in the form $\sqrt{A}+\sqrt{B}$, where A and B are integers and $A > B$. What is the value of $A - B$?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

10. What is the slope of the line that bisects the acute angle formed by the lines $y = (5/12)x$ and $y = (3/4)x$?

    (A) $\frac{1}{2}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{2}{3}$

11. Let A and B satisfy $\log_2(\log_4 A) = 1$ and $\log_4(\log_2 B) = 1$. Find the value of $\frac{1}{A}+\frac{1}{B}$.

    (A) 2 **(B)** 1 **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{16}$

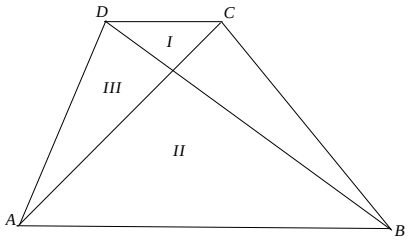
12. The value of $\sin 195^\circ + \sin 105^\circ$ is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** 1 **(E)** $\sqrt{2}$

13. The smallest positive integer which can be expressed both as a sum of ten consecutive positive integers and eleven consecutive positive integers is

(A) 55 **(B)** 110 **(C)** 130 **(D)** 165 **(E)** 275

14. In trapezoid $ABCD$, the area of region I is 9 and the area of region II is 16. What is the area of region III?



(A) 10 **(B)** 11 **(C)** 12 **(D)** 12.5 **(E)** cannot be determined

15. The polynomial

$$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

can be factored as $P_2(x) \cdot P_6(x)$ where P_2 and P_6 are polynomials with integer coefficients and have degrees 2 and 6 respectively. Find the sum of the coefficients of P_2.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

16. Given $\triangle ABC$ with $AB = AC = 10$, points D, E, and F lie on sides BC, AC, and AB respectively so that $BF = 5$, $EC = 4$, and the three segments AD, BE, and CF are concurrent. If this point of concurrency is named O, what is the ratio $AO : OD$?

(A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** $\frac{14}{5}$ **(D)** 3 **(E)** $\frac{7}{2}$

17. The sum of two of the roots of $p(x) = 4x^3 + 8x^2 - 9x - k$, where k is a constant, is zero. Find the value of k.

(A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 200

18. An urn contains two red, two blue, two white, and two yellow balls. Susan draws balls at random from the urn without replacing them. What is the expected number of draws Susan makes until drawing her first red ball?

(A) $\frac{42}{14}$ **(B)** $\frac{43}{14}$ **(C)** $\frac{42}{13}$ **(D)** $\frac{43}{13}$ **(E)** $\frac{44}{13}$

19. A five-digit integer, with all distinct digits which in this problem must be 1,2,3,4, and 5 in some order, is called *alternating* if the digits alternate between increasing and decreasing in size as read from left to right. They may start on an increasing or decreasing foot. For instance, both 34152 and 53412 are alternating while 12354 is not, for example. How many of this kind of 5 digit integer are alternating?

(A) 32 **(B)** 28 **(C)** 24 **(D)** 20 **(E)** 16

20. In $\triangle ABC$, $AB = 5$, $BC = 6$, and $CA = 4$. Side BC is trisected by points P and Q. Determine the value of $(AP)^2 + (AQ)^2$.

(A) $7\sqrt{7}$ **(B)** 25 **(C)** 29 **(D)** $11\sqrt{7}$ **(E)** 33