Fall 2011 McNabb GDCTM Contest
Pre-Calculus

NO Calculators Allowed

1. The expression
$$3\big((a+3b)4+2(5b+a)\big)$$
is equivalent to the expression

(A) $18a+22b$ **(B)** $15a+22b$ **(C)** $42a+42b$
(D) $15a+66b$ **(E)** $18a+66b$

2. If a raindrop has a volume of 10 cubic millimeters, a certain school yard has dimensions 50 meters by 40 meters, and this yard receives 5 centimeters of rain, the number of raindrops that fell on the yard is

(A) 10^9 **(B)** 10^{10} **(C)** 10^{11} **(D)** 10^{12} **(E)** 10^{13}

3. Recall that $[x]$ denotes the greatest integer less than or equal to x. If $f(x) = [x^2] - [x]^2$, find $f(\pi)$.

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

4. How many subsets of $\{a,b,c,d,e\}$ have an odd number of elements?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

5. How many perfect squares are in the sequence of integers
$$1, 11, 111, 1111, 11111, 111111, \cdots$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

6. Suppose $a+b^{-1}=4$ and $b+a^{-1}=4/3$. If $a>b^{-1}$ what is the value of ab?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

7. If $f(3x+1) = \dfrac{2}{x+4}$, then $f(x+3) =$

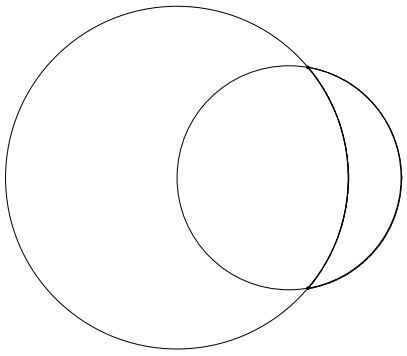
(A) $\dfrac{6}{x+14}$ **(B)** $\dfrac{6}{3x+14}$ **(C)** $\dfrac{6}{x+11}$ **(D)** $\dfrac{6}{x+17}$ **(E)** $\dfrac{2}{x+14}$

8. Independently of each other, Hezy and Zeke each randomly pick a real number between 0 and 3. What is the probability that their choices differ by at least 1?

(A) $1/9$ **(B)** $1/3$ **(C)** $1/2$ **(D)** $4/9$ **(E)** $5/9$

9. The distance between the two centers of the two circles is 3, the center of the larger circle lies on the smaller circle, and the points of intersection of the two circles lie on the same diameter of the smaller circle. Find the area of that part of the smaller circle that lies outside of the larger circle.

(A) 9 **(B)** $\pi/4$ **(C)** $9\pi/8 + 3\sqrt{2}/2$ **(D)** $3\pi - 3$ **(E)** 10



10. What is the positive difference between the largest and smallest real solutions of

$$x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. Let $f(x) = ax^2 + bx + c$, where a, b, and c are all non-zero constants. If $c = \dfrac{b^2}{4a}$, then the graph of f must

(A) be symmetric with respect to the y axis
(B) be symmetric with respect to the x axis
(C) be tangent to the x axis
(D) be tangent to the y axis
(E) have a maximum point

12. In acute triangle ABC, the intersection of its three altitudes, called the *orthocenter*, is labeled P. Given that $AP = 6$, $BP = 4$, and $BC = 10$, find AC.

(A) $\sqrt{120}$ **(B)** $\sqrt{130}$ **(C)** $\sqrt{140}$ **(D)** $\sqrt{150}$ **(E)** $\sqrt{160}$

13. A certain polynomial $P(x)$ has the property that all its coefficients are non-negative integers, none of which is larger than 6. If $P(7) = 2011$, what is P's coefficient of x^2?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. Square $ABCD$ of side-length 2 is inscribed in a circle. If chord AP bisects segment BC of the square, what is the square of the length of chord AP?

(A) 7 **(B)** 7.2 **(C)** 7.4 **(D)** 7.5 **(E)** 7.7

15. Five horses are in a race. In how many ways can they finish if ties are allowed?

(A) 511 **(B)** 530 **(C)** 531 **(D)** 541 **(E)** 625

16. Find the coefficient of x^{30} in the expansion of

$$(1 + x^3)(1 + x^6)(1 + x^9)(1 + x^{12}) \cdots (1 + x^{27})(1 + x^{30})$$

(A) 9 **(B)** 10 **(C)** 33 **(D)** 43 **(E)** 57

17. The graph of $y = ax^3 + bx^2 + cx + d$ is shown below. Which of the following must be true?

I. $a < 0$

II. $c < 0$

III. $cd > 0$

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** I,II, and III

18. Find the radius of the sphere that contains the points $(0, -3, -2)$, $(0, -3, 2)$, $(2, 3, 1)$, and $(-2, 3, 1)$.

(A) $\frac{43}{12}$ **(B)** $\frac{\sqrt{1945}}{12}$ **(C)** $\frac{47}{12}$ **(D)** $\frac{\sqrt{2011}}{12}$ **(E)** $\frac{49}{12}$

19. How many solutions are there to the equation $(1.1)^x = \log_{1.1} x$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

20. If $x^4 + \frac{1}{x^4} = 194$, then $x^6 + \frac{1}{x^6}$ could be equal to

(A) 2702 **(B)** 2730 **(C)** 3088 **(D)** 3090 **(E)** 3102