# Spring 2014 GDCTM/McNabb PreCalculus Contest

NO Calculators Allowed

1. A baseball is hit so that its height in feet t seconds after impact is given by $h(t) = 80t - 16t^2 + 3$. What is the maximum height, in feet, this baseball reaches?

2. If $\sec x - \tan x = 2$ find the value of $\sec x + \tan x$.

3. Factor $x^4 + 2x^3 - 15x^2 + 8x - 1$ into the product of two quadratic polynomials with integer coefecients.

4. Eight cows graze a pristine field bare in 40 days. It would take 15 cows just 12 days to graze the same pristine field bare. How many days would it take 10 cows to graze that same pristine field bare? Assume that the grass in this field grows at a constant rate and the cows graze at a constant rate.

5. If $x \ln x = 15$ then what is the least integer greater than x?

6. Solve the equation
$$\sqrt[3]{x+8} + \sqrt[3]{8-x} = 1$$

7. The graph of the parametric curve $x = a\cos bt$, $y = \sin 3t$ for $0 \le t < 2\pi$ is shown below. If a and b are positive integers find the value of ab.
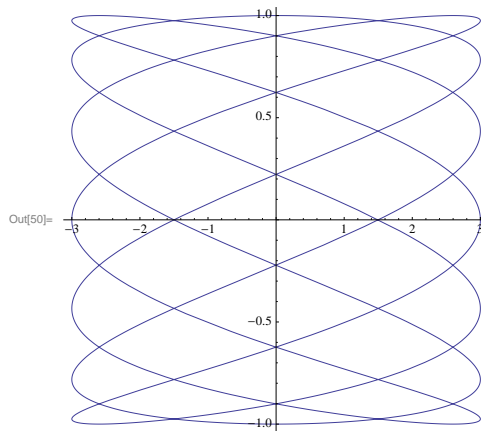
8. If x solves
$$\frac{1}{x} = \frac{1}{x+1} + \frac{1}{x+2014}$$
find the value of x^2.

9. Find the period of the function
$$f(x) = \frac{\cos(x/3) + \sec(x/2) + \cot(x/4)}{\sin(x/5) + \tan(x/8) + \csc(x/9)}$$

10. Let $a_1, a_2, a_3, \cdots$ be an infinite geometric sequence. Suppose that $\sum_{n=1}^{\infty} a_n = -9/20$ and $\sum_{n=4}^{\infty} a_n = 2/15$. Find the common ratio of this sequence.

11. At what point (x, y) in the coordinate plane does the curve $(t^3 - 16t, t^2 - 4)$ intersect itself?

12. Find the range of the function
$$f(x) = \frac{4}{x-4} - \frac{9}{x-9}$$
Give your answer in interval notation.

13. Quadrilateral $ABCD$ is inscribed in a circle with $AB = 5$, $BC = 7$, $CD = 6$ and $DA = 8$. Find the ratio AC/BD.

14. Let z and w be complex numbers satisfying $|z| = 1$ and $|w - 3 - 4i| = 1$. Find the maximum possible value of $|z + w|$.

15. A data center has 3 distinct servers, labeled A, B, and C. Each web request is routed independently to one of these servers. A request is routed to A with probability $1/2$, to B with probability $1/3$, and to C with probability $1/6$. What is the expected number of web requests that are received immediately before the first request that is routed to C?