Fall 2016 McNabb GDCTM Contest
## PreCalculus

NO Calculators Allowed

1. On a number line, the number $4/5$ ths of the way from a to $8/9$ is $7/9$. Find $3a$.

2. In parallelogram $ABCD$, diagonals AC and BD intersect at point E. If $\angle BAC = 70°$ and $\angle BDC = 40°$, find $\angle AEB$.

3. How many zero's occur when the number $2^{23} * 3 * 5^{24} * 7$ is written out in standard form?

4. Simplify to standard complex number form:
$$\frac{1}{i} + \frac{2}{i+1}$$

5. Suppose that $ABCDEFGH$ is a regular octagon of side length 1. What is the area of quadrilateral $ACEG$?
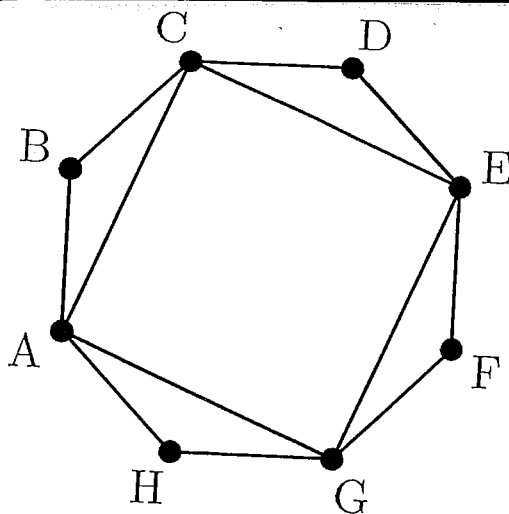
6. Find
$$\sin(1°) \cdot \sin(2°) \cdot \sin(3°) \cdot ... \cdot \sin(179°)$$

7. Solve the equation
$$\log_2(x) - \log_4(x+1) = 1$$
for x.

8. Find positive integers x, y such that $x^2 - 10y^2 = 1$

9. How many ordered triples of positive odd integers (m, n, p) satisfy

$$m + n + p = 21$$

?

10. Let $f(x) = \dfrac{3x + 8}{2x + 5}$ and $g(x) = \dfrac{3 + 2x}{3x}$. Find the value of $(f \circ g^{-1})^{-1} \left(\dfrac{11}{7} \right)$.

11. Find three odd positive integers a, b, and c, which satisfy

$$a^2 + b^2 + c^2 = 3abc$$
$$a + b + c = 19$$

12. Find two positive rational numbers r and s, neither of which are integers, so that $r^2 + s^2 = 17$.

13. Find all the real and complex roots of the polynomial

$$(x + 1)^5 + (x + 1)^4(x - 1) + (x + 1)^3(x - 1)^2 +$$
$$+ (x + 1)^2(x - 1)^3 + (x + 1)(x - 1)^4 + (x - 1)^5$$

14. A sphere is inscribed in a cone. The ratio of the height of the cone to the radius of the cone is known to be $\sqrt{3}$. What fraction of the volume of the cone is taken up by the sphere?

15. Compute the sum

$$\sum_{k=0}^{\infty} \frac{k^2 - k}{2^{k+1}}$$