Spring 2019 McNabb GDCTM Contest
Pre-Calculus

NO Calculators Allowed/ 60 Minutes

Assume all variables are real unless otherwise stated in the problem.

1. A store raised the price of a dozen eggs from \$2.50 to \$2.85. What was the percent increase?

2. Find one ordered pair of positive integers (m, n) which satisfies the equation $m^2 - 5n^2 = 1$.

3. Let $i = \sqrt{-1}$. Simplify $(1 + i)^3$.

4. Let $\sin\theta = 8/17$ and $\cos\theta = 15/17$. Find the value of $\tan 2\theta$.

5. Let $p(x) = x^4 - 2x^3 + 3x^2 - 2x + 5$. Find the sum of the coefficients of $p(2x)$.

6. Let
$$f(x) = \begin{cases} 2x + 4 & \text{if } x \le -2 \\ \dfrac{1}{2}x + 1 & \text{if } x > 2 \end{cases}$$
Find the value of $f^{-1}(7)$.

7. Define the sum of two points $P(a, b)$ and $Q(c, d)$ in the plane to be as if they were vectors, namely, $P + Q = (a + c, b + d)$. Let S and T be two regions in the plane. Define their sum as
$$S + T = \{P + Q \mid P \in S,\ Q \in T\}$$
Let
$$S = \{(x, y) \mid x^2 + y^2 \le 1\}$$
$$T = \{(x, y) \mid 4 \le x \le 5 \quad \text{and} \quad 0 \le y \le 1\}$$
Find the area of $S + T$.

8. Simplify
$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}^8$$

9. Consider an infinite geometric series consisting of positive terms such that the sum of the first three terms is 39 and the sum of the reciprocals of the first three terms is 13/27. Find the sum of this infinite series.

10. Find the number of ways to color the edges of a square if four colors are available and two colorings are considered the same if one can be rotated into the other.

11. How many paths are there from the point $(0, 0, 0, 0)$ to the point $(2, 2, 2, 2)$ if the only possible moves are to increase just a single coordinate by 1?

12. Solve
$$6(\log_8 x) \cdot (\log_2 x) + 6 \log_4 x = -1$$

13. Let x be an angle measured in radians such that $0 < x < \pi/2$ and

$$\cos x = \frac{1}{2} \cdot \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

Find the value of x.

14. Let z stand for a complex number. Find the area of the region in the complex plane which consists of all z such that

$$|z - 2|^2 + |z + 2|^2 \leq 26$$

15. Completely factor the polynomial

$$a^2(b + c) + b^2(c + a) + abc - c^3$$