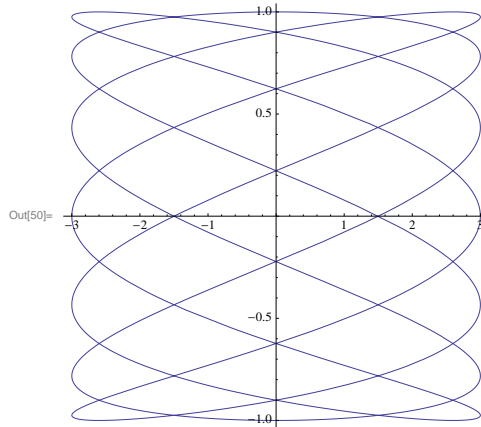


SPRING 2014 GDCTM/McNABB CALCULUS CONTEST

NO Calculators Allowed

1. The graph of the parametric curve $x = a \cos bt$, $y = \sin 3t$ for $0 \leq t < 2\pi$ is shown below. If a and b are positive integers find the value of ab .



2. If x solves

$$\frac{1}{x} = \frac{1}{x+1} + \frac{1}{x+2014}$$

find the value of x^2 .

3. Find the period of the function

$$f(x) = \frac{\cos(x/3) + \sec(x/2) + \cot(x/4)}{\sin(x/5) + \tan(x/8) + \csc(x/9)}$$

4. Let a_1, a_2, a_3, \dots be an infinite geometric sequence. Suppose that $\sum_{n=1}^{\infty} a_n = -9/20$ and $\sum_{n=4}^{\infty} a_n = 2/15$. Find the common ratio of this sequence.

5. Find the range of the function

$$f(x) = \frac{4}{x-4} - \frac{9}{x-9}$$

Give your answer in interval notation.

6. A data center has 3 distinct servers, labeled A , B , and C . Each web request is routed independently to one of these servers. A request is routed to A with probability $1/2$, to B with probability $1/3$, and to C with probability $1/6$. What is the expected number of web requests that are received immediately before the first request that is routed to C ?

7. Let f be a continuous, strictly increasing function. If $\int_1^9 f(x) dx = 52$, $f(1) = 4$, and $f(9) = 8$, find the value of $\int_4^8 f^{-1}(x) dx$.

8. Find

$$\lim_{a \rightarrow 0} \frac{\int_0^a \ln(1+ax) dx}{a^3}$$

9. Let $f(x)$ be a function integrable on the interval $[-5, 5]$. If $\int_{-5}^3 f(x) dx = 8$ and $\int_0^3 f(x) dx = 1$, find the value of $\int_{-5}^0 3f(x) dx$.

10. Let g be a differentiable function on the interval $[-3, 3]$ so that $g(1) = 0$. If

$$f(x) = \int_0^{g(x)} \frac{1}{t^4 + 4} dt$$

find the value of $\lim_{x \rightarrow 1} \frac{g(x)}{f(x)}$.

11. Let $F(a) = \int_0^a x^3 - ax \, dx$ for $a \geq 0$. Find the minimum value of $F(a)$.

12. Evaluate the integral

$$\int_0^{\pi/3} \sin(5x) \sin(3x) \, dx$$

13. Define recursively the sequences x_n and y_n by

$$\begin{aligned} x_{n+1} &= \frac{1}{3}x_n + \frac{2}{3}y_n \\ y_{n+1} &= \frac{2}{3}x_n + \frac{1}{3}y_n \end{aligned}$$

If $x_1 = 30$ and $y_1 = 40$ find $\lim_{n \rightarrow \infty} (x_n, y_n)$.

14. Let b be a real number and consider the cubic equation $x^3 + 3bx + 3 = 0$. The set of all real numbers b so that this cubic has three distinct real solutions is an interval of the form $(-\infty, r)$.

The real number r has the form $r = -\sqrt[3]{\frac{a}{b}}$ where a and b are positive integers with no common factor greater than one. Find the value of $a + b$.

15. Evaluate the integral

$$\int_0^1 \frac{x^{\sqrt{2}} - 1}{\ln x} \, dx$$