

SPRING 2019 McNABB GDCTM CONTEST
CALCULUS

NO Calculators Allowed/ 60 Minutes

Assume all variables are real unless otherwise stated in the problem.

1. The angle bisectors of the angles of $\triangle ABC$ meet at point D . If $\angle A = 100^\circ$, find the measure of $\angle BDC$ in degrees.
2. What is the number of positive factors of the number 20^{19} ?
3. Find the value of the limit
$$\lim_{n \rightarrow \infty} \frac{3n + 2}{2n + 3}$$
4. If $\cos 2\theta = 1/9$, find the least possible value of $\sin \theta$.
5. Find the equation of the tangent line to $x^3 + y^3 = \frac{28xy}{3}$ if the point of tangency is $(3, 1)$. Answer in form $y = mx + b$.
6. For what values of the constant a does $y = \sin(ax)$ solve the differential equation $y'' + 9y = 0$?
7. Let $f(x)$ be continuous on the interval $[0, 5]$. If $\int_0^4 f(x) dx = 3$ and $\int_4^5 f(x) dx = -7$, find the value of $\int_0^5 f(x) dx$
8. Find the value of $\int_0^{\pi/6} \sin 2x \cos 3x dx$
9. Let $f(x)$ be differentiable on the interval $[0, 1]$. If $\int_0^1 xf(x) dx = 3$ and $\int_0^1 x^2 f'(x) dx = 4$, then find the value of $f(1)$.
10. Find the value of $\int_2^4 \frac{2x}{x^4 - 1} dx$
11. A spherical solid of radius 6 cm has density $\rho(r) = 10/r$ gm/cm³, where r is the distance to the center of the sphere in cm. What is the mass in grams of this solid?
12. For what value of the real parameter a does the polynomial $x^4 - 3x^3 - 6x^2 + ax - 24$ have a double root?

13. Determine the values of the real-valued parameters a and b which minimize $\int_0^1 (x^2 - ax - b)^2 dx$.
Then put in the answer box the number $a + b$.
14. Let $F(x, y) = x^2 + y^2$ and $\Omega = \{(x, y) \mid x^2 + y^2 \leq 1\}$. Find the average value of $F(x, y)$ over the set Ω .
15. Find all values of the real parameter a such that the cubic $x^3 - 2x^2 + x + a$ has only real roots.
Answer in interval notation form.