Fall 2011 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

1. A certain number is doubled. The result is then increased by nine. This result is decreased by 3. If this last number is 28, what was the original number?

   (A) -4 **(B)** 0 **(C)** 7 **(D)** 11 **(E)** 28

2. Two sweaters, a pair of wool socks, and a coat cost \$180. One sweater and the coat cost \$ 130. How much does one sweater and a pair of wool socks cost?

   (A) \$30 **(B)** \$40 **(C)** \$50 **(D)** \$60 **(E)** \$70

3. The expression

   $$3\big((a+3b)4+2(5b+a)\big)$$

   is equivalent to the expression

   (A) $18a+22b$ **(B)** $15a+22b$ **(C)** $42a+42b$
   (D) $15a+66b$ **(E)** $18a+66b$

4. How many subsets of $\{a,b,c,d,e\}$ have an odd number of elements?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

5. From a regular deck of 52 cards three cards are dealt to you. What is the probability all three are red cards? Recall the red suites are hearts and diamonds.

   (A) $2/17$ **(B)** $1/8$ **(C)** $2/15$ **(D)** $1/7$ **(E)** $2/13$

6. Xenia is three years older than Zyler. Eight years ago Zyler was half the age of Xenia. How many years from now will Xenia be $8/7$ ths the age of Zyler?

   (A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

7. On the real number line, let point A have coordinate a and point B have coordinate b. What is the coordinate of the point between A and B which is four times closer to B than it is to A?

(A) $\frac{4a+b}{5}$ **(B)** $\frac{3a+b}{4}$ **(C)** $\frac{a+4b}{5}$ **(D)** $\frac{a+3b}{4}$ **(E)** $\frac{a+b}{2}$

8. The number of solutions of the equation

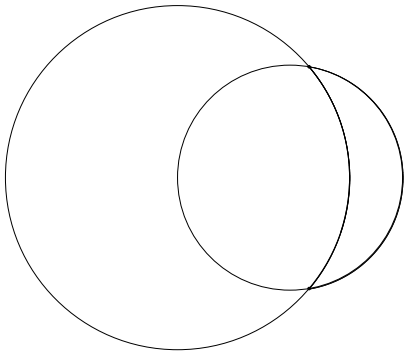
$$|x-1|+|x-2|=|x-3|$$

is equal to

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. The distance between the two centers of the two circles is 3, the center of the larger circle lies on the smaller circle, and the points of intersection of the two circles lie on the same diameter of the smaller circle. Find the area of that part of the smaller circle that lies outside of the larger circle.

(A) 9 **(B)** $\pi/4$ **(C)** $9\pi/8+3\sqrt{2}/2$ **(D)** $3\pi-3$ **(E)** 10



10. If $a<b<c<d<e$ which of the following must be true?

(A) $ab<cd$
(B) $c-a<e-c$
(C) $a^2<e^2$
(D) $ad+bc<ac+bd$
(E) $b+d<2c$

11. If a raindrop has a volume of 10 cubic millimeters, a certain school yard has dimensions 50 meters by 40 meters, and this yard receives 5 centimeters of rain, the number of raindrops that fell on the yard is

(A) 10^9 **(B)** 10^{10} **(C)** 10^{11} **(D)** 10^{12} **(E)** 10^{13}

12. Find the sum of all the factors of 280.

(A) 440 **(B)** 540 **(C)** 600 **(D)** 640 **(E)** 720

13. The expression

$$a - (b + (c - (d + (e - f))))$$

is equivalent to

(A) $a - b - c + d + e - f$
(B) $a - b - c + d - e - f$
(C) $a - b - c + d + e + f$
(D) $a - b + c - d + e - f$
(E) $a - b + c - d - e - f$

14. The product of the repeating decimals $0.\overline{3}$ and $0.\overline{12}$ is

(A) $0.\overline{03}$ **(B)** $0.\overline{04}$ **(C)** $0.\overline{36}$
(D) $0.\overline{6}$ **(E)** not repeating

15. Amanda, Brice, and Carl all start working at SellMore with the same salary on the same day. They receive the following percent raises in their salary at the end of each of the first two years in order:

Amanda: 5%, 3%

Brice: 3%, 5%

Carl: 4%, 4%

Which of them earns the most total over their first three years at SellMore?

(A) Amanda **(B)** Brice **(C)** Carl
(D) Amanda and Brice tied for the most **(E)** All three tied for the most

16. What is the smallest 4 digit prime number?

(A) 1001 **(B)** 1003 **(C)** 1005 **(D)** 1007 **(E)** 1009

17. If p people consume m pounds of mashed potato in h hours, then the pounds of mashed potato consumed by m people in p hours equals:

(A) mph **(B)** $\frac{m}{ph}$ **(C)** $\frac{m^2}{ph}$ **(D)** $\frac{m^2}{h}$ **(E)** $\frac{p^2}{m}$

18. If a positive integer n has exactly 12 factors, what is the difference between the greatest and least number of factors that n^2 could have?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 27

19. Hezy and Zeke have a 6 hour drive to get to their grandparents house for Thanksgiving. Each will drive on their turn(s), if they have a turn, a positive whole number of hours. They can switch drivers or not as they wish, so long as they follow the rule of each driver driving a whole number of hours on their turn(s). They could even not switch at all. If Hezy starts the trip, in how many different ways of sharing (or not!) the driving, can they get to their grandparents?

(A) 12 **(B)** 24 **(C)** 30 **(D)** 32 **(E)** 64

20. Blindfolded, Sue rolls two standard cubical dice. Her friend tells her that the sum of the two numbers rolled is less than six. What is the probability that Sue rolled snake-eyes, that is, two ones?

(A) 1/36 **(B)** 1/18 **(C)** 1/12 **(D)** 1/11 **(E)** 1/10

21. How many of the numbers in this set below are irrational?

$$\{\sqrt{1.00}, \sqrt{1.01}, \sqrt{1.02}, \sqrt{1.03}, \cdots, \sqrt{3.98}, \sqrt{3.99}\}$$

(A) 299 **(B)** 294 **(C)** 290 **(D)** 286 **(E)** 150

22. Find the sum

$$1 \cdot 25 + 2 \cdot 24 + 3 \cdot 23 + 4 \cdot 22 + \cdots + 24 \cdot 2 + 25 \cdot 1$$

(A) 2500 **(B)** 2725 **(C)** 2800 **(D)** 2825 **(E)** 2925

23. How many factors of $51^5 \cdot 71^7 \cdot 91^9$ are perfect squares?

(A) 1 **(B)** 60 **(C)** 180 **(D)** 192 **(E)** 900

24. If $a = \dfrac{1110}{1111}$, $b = \dfrac{2221}{2223}$, and $c = \dfrac{3331}{3334}$ which of the following is true?

(A) $a > b > c$ **(B)** $b > a > c$ **(C)** $c > a > b$
(D) $c > b > a$ **(E)** $b > c > a$

25. In how many ways can the letters in the word *monsoon* be arranged so that the second n occurs before the third o?

(A) 210 **(B)** 216 **(C)** 252 **(D)** 256 **(E)** 260

Fall 2011 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

1. A certain number is doubled. The result is then increased by nine. This result is decreased by 3. If this last number is 28, what was the original number?

   (A) -4 **(B)** 0 **(C)** 7 **(D)** 11 **(E)** 28

2. The expression

$$3\big((a+3b)4+2(5b+a)\big)$$

   is equivalent to the expression

   (A) $18a+22b$ **(B)** $15a+22b$ **(C)** $42a+42b$
   (D) $15a+66b$ **(E)** $18a+66b$

3. How many of the numbers in this set below are irrational?

$$\sqrt{1.00}, \sqrt{1.01}, \sqrt{1.02}, \sqrt{1.03}, \cdots, \sqrt{3.98}, \sqrt{3.99}$$

   (A) 299 **(B)** 294 **(C)** 290 **(D)** 286 **(E)** 150

4. If $f(3x+1)=\dfrac{2}{x+4}$, then $f(x+3)=$

   (A) $\dfrac{6}{x+14}$ **(B)** $\dfrac{6}{3x+14}$ **(C)** $\dfrac{6}{x+11}$ **(D)** $\dfrac{6}{x+17}$ **(E)** $\dfrac{2}{x+14}$

5. An ordered pair (m,n) of positive integers is called a *three-pair* if the interior angle of a regular polygon with m sides is three times the exterior angle of a regular polygon with n sides. How many *three-pairs* exist?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** more than 6

6. Amanda, Brice, and Carl all start working at SellMore with the same salary on the same day. They receive the following percent raises in their salary at the end of each of the first two years in order:

   Amanda: 5%, 3%

   Brice: 3%, 5%

   Carl: 4%, 4%

   Which of them earns the most total over their first three years at SellMore?

   (A) Amanda **(B)** Brice **(C)** Carl
   (D) Amanda and Brice tied for the most **(E)** All three tied for the most

7. Let a, b, c, and d be non-zero constants. If the lines $ax + by = 0$ and $cx + dy = 0$ are perpendicular then which of these quantities must be zero?

   (A) $ad - bc$ **(B)** $ac + bd$ **(C)** $ac - bd$
   (D) $ad + bc$ **(E)** $a^2 + b^2 + c^2 + d^2$

8. If p people consume m pounds of mashed potato in h hours, then the pounds of mashed potato consumed by m people in p hours equals:

   (A) mph **(B)** $\frac{m}{ph}$ **(C)** $\frac{m^2}{ph}$ **(D)** $\frac{m^2}{h}$ **(E)** $\frac{p^2}{m}$

9. If $a < b < c < d < e$ which of the following must be true?

   (A) $ab < cd$
   (B) $c - a < e - c$
   (C) $a^2 < e^2$
   (D) $ad + bc < ac + bd$
   (E) $b + d < 2c$

10. Find the coefficient of x^{30} in the expansion of

$$(1 + x^3)(1 + x^6)(1 + x^9)(1 + x^{12}) \cdots (1 + x^{27})(1 + x^{30})$$

    (A) 9 **(B)** 10 **(C)** 33 **(D)** 43 **(E)** 57

11. In how many ways can the letters in the word *monsoon* be arranged so that the second n occurs before the third o?

(A) 210 **(B)** 216 **(C)** 252 **(D)** 256 **(E)** 260

12. Suppose $a + b^{-1} = 4$ and $b + a^{-1} = 4/3$. If $a > b^{-1}$ what is the value of ab?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

13. In $\triangle ABC$, points E and F lie on $\overline{BC}$ and $\overline{AC}$ respectively. Let $\overline{AE}$ and $\overline{BF}$ intersect at G. If $\dfrac{AF}{FC} = \dfrac{3}{5}$ and G is the midpoint of $\overline{BF}$, then find the ratio $\dfrac{CE}{EB}$.

(A) 3 **(B)** $7/2$ **(C)** $8/3$ **(D)** $3/2$ **(E)** $11/4$

14. Independently of each other, Hezy and Zeke each randomly pick a real number between 0 and 3. What is the probability that their choices differ by at least 1?

(A) $1/9$ **(B)** $1/3$ **(C)** $1/2$ **(D)** $4/9$ **(E)** $5/9$

15. Find the area enclosed by the graph of

$$|2y - 1| + |2y + 1| + 2|x| = 4$$

(A) 2 **(B)** 2.5 **(C)** 3 **(D)** 3.5 **(E)** 4

16. Find the sum

$$1 \cdot 25 + 2 \cdot 24 + 3 \cdot 23 + 4 \cdot 22 + \cdots + 24 \cdot 2 + 25 \cdot 1$$

(A) 2500 **(B)** 2725 **(C)** 2800 **(D)** 2825 **(E)** 2925

17. Square $ABCD$ of side-length 2 is inscribed in a circle. If chord AP bisects segment BC of the square, what is the square of the length of chord AP?

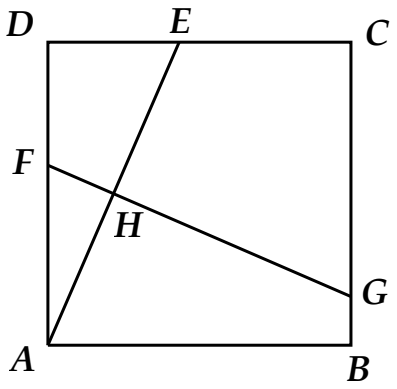
(A) 7 **(B)** 7.2 **(C)** 7.4 **(D)** 7.5 **(E)** 7.7

18. Let $f(x) = ax^2 + bx + c$, where a, b, and c are all non-zero constants. If $c = \dfrac{b^2}{4a}$, then the graph of f must

(A) be symmetric with respect to the y axis
(B) be symmetric with respect to the x axis
(C) be tangent to the x axis
(D) be tangent to the y axis
(E) have a maximum point

19. Inside square $ABCD$ point E lies on side $\overline{CD}$ with $\dfrac{CE}{ED} = \dfrac{5}{3}$. The perpendicular bisector of $\overline{AE}$ intersects the square at points F and G and intersects $\overline{AE}$ at H, as shown. Find the ratio $\dfrac{FH}{HG}$

(A) $3/13$ **(B)** $1/4$ **(C)** $2/7$ **(D)** $1/3$ **(E)** $4/11$



20. In acute triangle ABC, the intersection of its three altitudes, called the *orthocenter*, is labeled P. Given that $AP = 6$, $BP = 4$, and $BC = 10$, find AC.

(A) $\sqrt{120}$ **(B)** $\sqrt{130}$ **(C)** $\sqrt{140}$ **(D)** $\sqrt{150}$ **(E)** $\sqrt{160}$

Fall 2011 McNabb GDCTM Contest
Calculus

NO Calculators Allowed

All variables are assumed to represent real numbers unless stated in the problem otherwise.

1. The number of even factors of 7^7 is

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2. Recall that $[x]$ denotes the greatest integer less than or equal to x. If $f(x) = [x^2] - [x]^2$, find $f(\pi)$.

   (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

3. How many subsets of $\{a, b, c, d, e\}$ have an odd number of elements?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

4. How many solutions are there to the equation $2^a = a^2$?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

5. Given that the piecewise function

$$f(x) = \begin{cases} 4x & \text{if } x \leq 0 \\ ax^2 + bx + c & \text{if } 0 < x < 1 \\ 6 - 3x & \text{if } x \geq 1 \end{cases}$$

   is differentiable at all real numbers, find the value of $a + b + c$.

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. The value of $13\sin(\tan^{-1}(5/12)) + 15\sin(\tan^{-1}(9/12))$ is

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

7. Let $f(x)$ be differentiable at $x = 3$. If $f(3) = 5$ and $\left(\frac{1}{f}\right)'(3) = 4$ then what is the value of $f'(3)$?

(A) -100 **(B)** -20 **(C)** -1/4 **(D)** 1/4 **(E)** 20

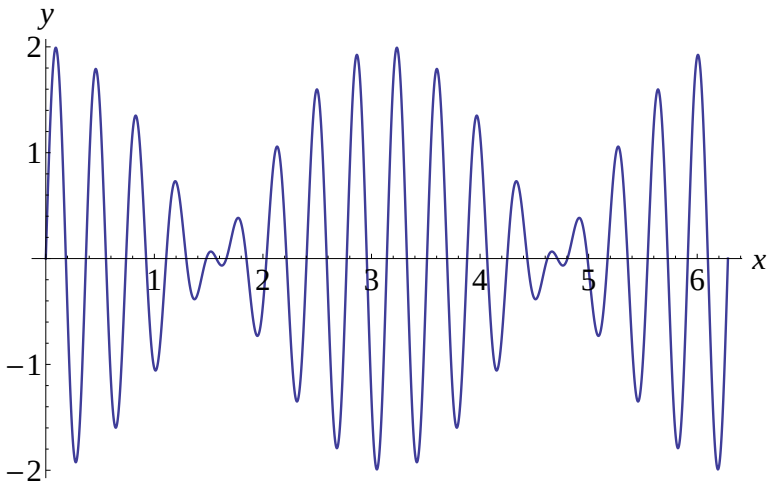
8. Determine the value of

$$\lim_{x \to 0} (\cos x)^{\frac{1}{\sin^2 x}}$$

(A) 0 **(B)** 1 **(C)** e **(D)** e^2 **(E)** $e^{-\frac{1}{2}}$

9. The graph of $y = \sin ax + \sin bx$ is shown below for x in the interval $[0, 2\pi]$. Given that a and b are positive integers, with $a + b$ large compared to $a - b$ and $a > b$, the value of $a - b$ could be:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7



10. An eight by eight matrix has its (i,j)th entry given by $f(i)g(j)$ where $f(i) = 4i - 3$ and $g(j) = 2j + 5$. What is the sum of all of the entries of this matrix?

(A) 13440 **(B)** 13540 **(C)** 13640 **(D)** 13740 **(E)** 13840

11. If $f(x) = x^7 + x$, then what is the value of the second derivative of the inverse function of f at 2? That is, what is $(f^{-1})''(2)$?

(A) -7/128 **(B)** -7/16 **(C)** -21/256 **(D)** -3/64 **(E)** 3/64

12. Find the absolute maximum value of $f(x) = \sin^8 x \cos^4 x$ on the interval $[0, 2\pi]$.

(A) $\frac{2^4}{3^6}$ **(B)** $\frac{1}{2^6}$ **(C)** $\frac{3^4}{2^{12}}$ **(D)** $\frac{3^6}{2^{10}}$ **(E)** $\frac{3^2}{2^6}$

13. Five horses are in a race. In how many ways can they finish if ties are allowed?

(A) 511 **(B)** 530 **(C)** 531 **(D)** 541 **(E)** 625

14. For the sequence given by $t_{n+1} = \frac{t_n + t_{n-1} + 1}{t_{n-2}}$, with $t_1 = 4$, $t_2 = 2$ and $t_3 = 5$, find t_{2011}.

(A) 4 **(B)** 2 **(C)** 5 **(D)** 7/5 **(E)** 16/5

15. Find the radius of the sphere that contains the points $(0, -3, -2)$, $(0, -3, 2)$, $(2, 3, 1)$, and $(-2, 3, 1)$.

(A) $\frac{43}{12}$ **(B)** $\frac{\sqrt{1945}}{12}$ **(C)** $\frac{47}{12}$ **(D)** $\frac{\sqrt{2011}}{12}$ **(E)** $\frac{49}{12}$

16. A certain polynomial $P(x)$ has the property that all its coefficients are non-negative integers, none of which is larger than 6. If $P(7) = 2011$, what is P's coefficient of x^2?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

17. If $a = \dfrac{1110}{1111}$, $b = \dfrac{2221}{2223}$, and $c = \dfrac{3331}{3334}$ which of the following is true?

(A) $a > b > c$ **(B)** $b > a > c$ **(C)** $c > a > b$ **(D)** $c > b > a$ **(E)** $b > c > a$

18. Which of the following are always true about the pair of functions $y = a^x$ and $y = \log_a x$, where $a > 1$?

I. Both are increasing on their domains

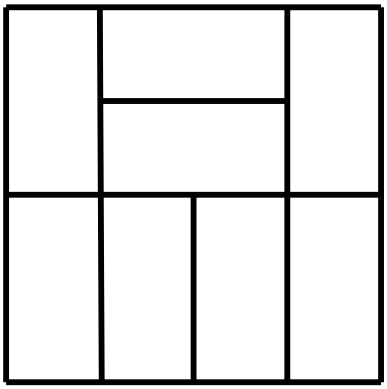
II. For the same $a > 1$, their graphs never intersect

III. For the same $a > 1$, they are inverses of each other.

(A) I only **(B)** II only **(C)** I and II only
(D) I and III only **(E)** I, II and III

19. In how many ways can a 4×4 nailed down board be tiled by eight 1×2 dominoes? One way to tile the board is shown below.

(A) 16 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 49



20. If $a + b + c = 9$ and $ab + bc + ca = 7$ then the maximum possible value of c is closest to

(A) 8 **(B)** 8.5 **(C)** 9 **(D)** 9.5 **(E)** 10

Fall 2011 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. Two sweaters, a pair of wool socks, and a coat cost \$180. One sweater and the coat cost \$ 130. How much does one sweater and a pair of wool socks cost?

   (A) \$30 **(B)** \$40 **(C)** \$50 **(D)** \$60 **(E)** \$70

2. The expression

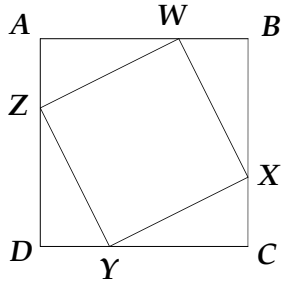
$$3\big((a+3b)4+2(5b+a)\big)$$

   is equivalent to the expression

   (A) $18a+22b$ **(B)** $15a+22b$ **(C)** $42a+42b$
   (D) $15a+66b$ **(E)** $18a+66b$

3. In square $ABCD$, square $WXYZ$ is inscribed in such a way that W is two-thirds of the way from A to B, X is two-thirds of the way from B to C, Y is two-thirds of the way from C to D, and Z is two-thirds of the way from D to A. If the area of $WXYZ$ is 100, what is the area of $ABCD$?

   (A) 150 **(B)** 160 **(C)** 170 **(D)** 180 **(E)** 200



4. From a regular deck of 52 cards three cards are dealt to you. What is the probability all three are red cards? Recall the red suites are hearts and diamonds.

   (A) $2/17$ **(B)** $1/8$ **(C)** $2/15$ **(D)** $1/7$ **(E)** $2/13$

5. If $f(3x+1) = \dfrac{2}{x+4}$, then $f(x+3) =$

(A) $\dfrac{6}{x+14}$ **(B)** $\dfrac{6}{3x+14}$ **(C)** $\dfrac{6}{x+11}$ **(D)** $\dfrac{6}{x+17}$ **(E)** $\dfrac{2}{x+14}$

6. A given cone's dimensions are modified as described in the responses below. Which response does **not** change the volume?

(A) double the height and halve the radius
(B) halve the height and double the radius
(C) quadruple the height and halve the radius
(D) halve the height and quadruple the radius
(E) quadruple the height and halve the radius twice

7. In how many ways can 10 be written as a sum of one or more positive integers if order does not matter and no integer can be repeated in a given sum? Thus, for instance, $4+6$ is considered the same as $6+4$, and $5+5$ is not allowed.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

8. Find the area enclosed by the graph of

$$|2y-1| + |2y+1| + 2|x| = 4$$

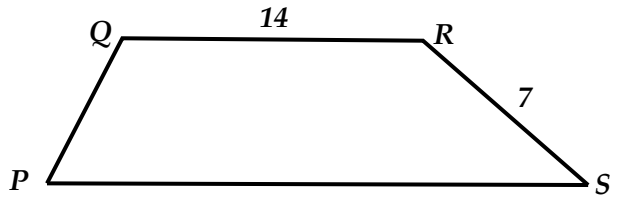
(A) 2 **(B)** 2.5 **(C)** 3 **(D)** 3.5 **(E)** 4

9. An ordered pair (m, n) of positive integers is called a *three-pair* if the interior angle of a regular polygon with m sides is three times the exterior angle of a regular polygon with n sides. How many *three-pairs* exist?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** more than 6

10. In trapezoid $PQRS$ as shown with $PS \parallel QR$, $QR = 14$, $RS = 7$, and $\angle R = 2\angle P$, the length of PS is

(A) 15 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 27



11. If p people consume m pounds of mashed potato in h hours, then the pounds of mashed potato consumed by m people in p hours equals:

(A) mph **(B)** $\frac{m}{ph}$ **(C)** $\frac{m^2}{ph}$ **(D)** $\frac{m^2}{h}$ **(E)** $\frac{p^2}{m}$

12. Find the sum

$$1 \cdot 25 + 2 \cdot 24 + 3 \cdot 23 + 4 \cdot 22 + \cdots + 24 \cdot 2 + 25 \cdot 1$$

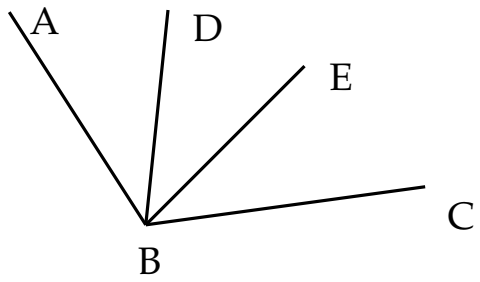
(A) 2500 **(B)** 2725 **(C)** 2800 **(D)** 2825 **(E)** 2925

13. How many of the numbers in this set below are irrational?

$$\{\sqrt{1.00}, \sqrt{1.01}, \sqrt{1.02}, \sqrt{1.03}, \cdots, \sqrt{3.98}, \sqrt{3.99}\}$$

(A) 299 **(B)** 294 **(C)** 290 **(D)** 286 **(E)** 150

14. $\angle ABC$ is trisected by rays $\overrightarrow{BD}$ and $\overrightarrow{BE}$ as shown. If the degree measure of $\angle DBC$ equals $3x - 5$ and that of $\angle ABC$ equals $5x - 22$, find the value of x.



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15. How many factors of $51^5 \cdot 71^7 \cdot 91^9$ are perfect squares?

(A) 1 **(B)** 60 **(C)** 180 **(D)** 192 **(E)** 900

16. In acute triangle ABC, the intersection of its three altitudes, called the *orthocenter*, is labeled P. Given that $AP = 6$, $BP = 4$, and $BC = 10$, find AC.

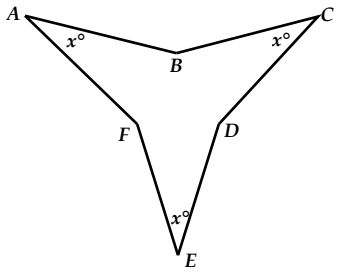
(A) $\sqrt{120}$ **(B)** $\sqrt{130}$ **(C)** $\sqrt{140}$ **(D)** $\sqrt{150}$ **(E)** $\sqrt{160}$

17. If $a = \dfrac{1110}{1111}$, $b = \dfrac{2221}{2223}$, and $c = \dfrac{3331}{3334}$ which of the following is true?

(A) $a > b > c$ **(B)** $b > a > c$ **(C)** $c > a > b$
(D) $c > b > a$ **(E)** $b > c > a$

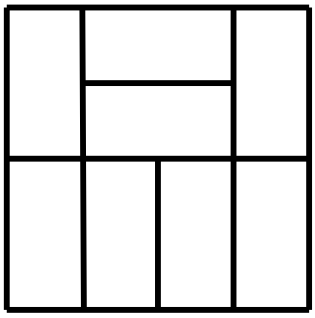
18. In non-convex hexagon $ABCDEF$, $AB = BC = CD = DE = EF = FA$ and $\angle A \cong \angle C \cong \angle E$. If the degree measure common to these three angles is x, what is the degree measure of $\angle ABC$ in terms of x?

(A) $120 + x$ **(B)** $180 - x$ **(C)** $90 + 2x$ **(D)** $180 - 3x$ **(E)** $60 + 3x$



19. In how many ways can a 4×4 nailed down board be tiled by eight 1×2 dominoes? One way to tile the board is shown below.

(A) 16 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 49



20. Let L_1 and L_2 be two intersecting lines. Let P be an arbitrary point of the plane determined by L_1 and L_2. Consider the following sequence of transformations in this plane. First, the point P is reflected across line L_1 to point Q. Second, point Q is reflected across line L_2 to point R. This sequence of transformations that maps point P to point R is equivalent to

(A) a translation
(B) a reflection about some third line
(C) a rotation about the point of intersection of the lines by an angle equal to the smaller angle formed by the lines
(D) a rotation about the point of intersection of the lines by an angle equal to twice the smaller angle formed by the lines
(E) a translation followed by a reflection about some third line

Fall 2011 McNabb GDCTM Contest
Pre-Algebra

NO Calculators Allowed

1. The sum
$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$$
is equal to

(A) 1/10 **(B)** 1/24 **(C)** 1/6 **(D)** 25/6 **(E)** 25/12

2. A certain number is doubled. The result is then increased by nine. This result is decreased by 3. If this last number is 28, what was the original number?

(A) -4 **(B)** 0 **(C)** 7 **(D)** 11 **(E)** 28

3. A train travels 1 mile in 1 minute and 20 seconds. At this speed, how many miles will the train travel in 112 minutes?

(A) 84 **(B)** 86 **(C)** 88 **(D)** 90 **(E)** 96

4. What is the number of square inches in a rectangle which measures $1\frac{1}{4}$ feet by $1\frac{1}{6}$ yards?

(A) 35/24 **(B)** 35/2 **(C)** 70 **(D)** 120 **(E)** 630

5. Two sweaters, a pair of wool socks, and a coat cost \$180. One sweater and the coat cost \$ 130. How much does one sweater and a pair of wool socks cost?

(A) \$30 **(B)** \$40 **(C)** \$50 **(D)** \$60 **(E)** \$70

6. Five siblings, each a different age, split a gift of \$200 in such a way that each child other than the youngest, gets ten dollars more than the next younger sibling. The youngest, of course, gets ten dollars less than the next to youngest. How much does the middle child receive?

(A) \$25 **(B)** \$30 **(C)** \$35 **(D)** \$40 **(E)** \$45

7. A given cone's dimensions are modified as described in the responses below. Which response does **not** change the volume?

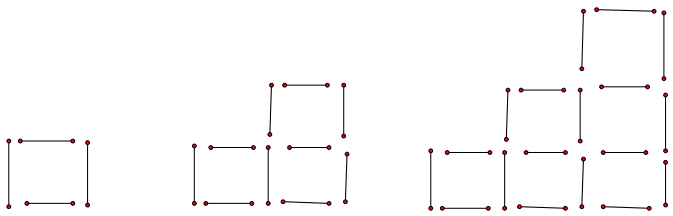
   (A) double the height and halve the radius
   (B) halve the height and double the radius
   (C) quadruple the height and halve the radius
   (D) halve the height and quadruple the radius
   (E) quadruple the height and halve the radius twice

8. The sum of the first n positive integers is 210. What is the average of these first n positive integers?

   (A) 9 **(B)** 9.5 **(C)** 10 **(D)** 10.5 **(E)** 21

9. In a sequence of of matchstick diagrams the next diagram adds one more column of blocks built one block higher and attached to the right of the previous diagram. Shown are the first three diagrams in this sequence. What is the number of matchsticks in the 7th diagram of this sequence?

   (A) 70 **(B)** 72 **(C)** 74 **(D)** 76 **(E)** 78



10. Amanda, Brice, and Carl all start working at SellMore with the same salary on the same day. They receive the following percent raises in their salary at the end of each of the first two years in order:

    Amanda: 5%, 3%

    Brice: 3%, 5%

    Carl: 4%, 4%,

    Which of them earns the most total over their first three years at SellMore?

    (A) Amanda **(B)** Brice **(C)** Carl
    (D) Amanda and Brice tied for the most **(E)** All three tied for the most

11. From a regular deck of 52 cards three cards are dealt to you. What is the probability all three are red cards? Recall the red suites are hearts and diamonds.

(A) $2/17$ **(B)** $1/8$ **(C)** $2/15$ **(D)** $1/7$ **(E)** $2/13$

12. The value of

$$\frac{1^3+2^3+3^3+4^3+\cdots+17^3}{4^3+8^3+12^3+16^3+\cdots+68^3}$$

is equal to

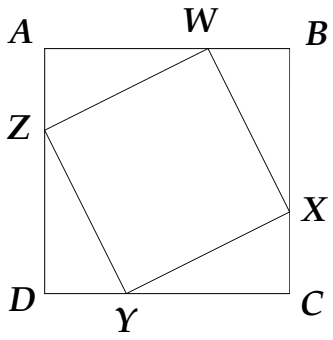
(A) $1/4$ **(B)** $1/16$ **(C)** $1/64$ **(D)** $1/68$ **(E)** $1/192$

13. In how many ways can 10 be written as a sum of one or more positive integers if order does not matter and no integer can be repeated in a given sum? Thus, for instance, $4+6$ is considered the same as $6+4$, and $5+5$ is not allowed.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

14. In square $ABCD$, square $WXYZ$ is inscribed in such a way that W is two-thirds of the way from A to B, X is two-thirds of the way from B to C, Y is two-thirds of the way from C to D, and Z is two-thirds of the way from D to A. If the area of $WXYZ$ is 100, what is the area of $ABCD$?

(A) 150 **(B)** 160 **(C)** 170 **(D)** 180 **(E)** 200



15. If a raindrop has a volume of 10 cubic millimeters, a certain school yard has dimensions 50 meters by 40 meters, and this yard receives 5 centimeters of rain, the number of raindrops that fell on the yard is

(A) 10^9 **(B)** 10^{10} **(C)** 10^{11} **(D)** 10^{12} **(E)** 10^{13}

16. If $n^3 = 18 \cdot 96$, then n^2 is equal to

(A) 36 **(B)** 81 **(C)** 121 **(D)** 144 **(E)** 196

17. Seven consecutive integers are written on a whiteboard. When one of them is erased, the sum of the remaining six integers is 857. What is the sum of the original seven integers?

(A) 1001 **(B)** 1008 **(C)** 1015 **(D)** 1085 **(E)** 1092

18. How many factors of $51^5 \cdot 71^7 \cdot 91^9$ are perfect squares?

(A) 1 **(B)** 60 **(C)** 180 **(D)** 192 **(E)** 900

19. Hezy and Zeke have a 6 hour drive to get to their grandparents house for Thanksgiving. Each will drive on their turn(s), if they have a turn, a positive whole number of hours. They can switch drivers or not as they wish, so long as they follow the rule of each driver driving a whole number of hours on their turn(s). They could even not switch at all. If Hezy starts the trip, in how many different ways of sharing (or not!) the driving, can they get to their grandparents?

(A) 24 **(B)** 30 **(C)** 32 **(D)** 64 **(E)** 120

20. In how many ways can a group of ten students be split into two groups of five each?

(A) 50 **(B)** 63 **(C)** 126 **(D)** 252 **(E)** $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

21. The product of the repeating decimals $0.\overline{3}$ and $0.\overline{12}$ is

(A) $0.\overline{03}$ **(B)** $0.\overline{04}$ **(C)** $0.\overline{36}$
(D) $0.\overline{6}$ **(E)** not repeating

22. Blindfolded, Sue rolls two standard cubical dice. Her friend tells her that the sum of the two numbers rolled is less than six. What is the probability that Sue rolled snake-eyes, that is, two ones?

(A) 1/36 **(B)** 1/18 **(C)** 1/12 **(D)** 1/11 **(E)** 1/10

23. Which of the following *cannot* be the number of zeros in which $n!$ ends?

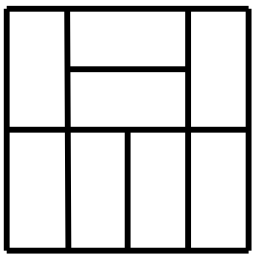
(A) 148 **(B)** 150 **(C)** 152 **(D)** 154 **(E)** 156

24. What is the smallest 4 digit prime number?

(A) 1001 **(B)** 1003 **(C)** 1005 **(D)** 1007 **(E)** 1009

25. In how many ways can a 4×4 nailed down board be tiled by eight 1×2 dominoes? One way to tile the board is shown below.

(A) 16 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 49



Fall 2011 McNabb GDCTM Contest
Pre-Calculus

NO Calculators Allowed

1. The expression
$$3\big((a+3b)4+2(5b+a)\big)$$
is equivalent to the expression

(A) $18a+22b$ **(B)** $15a+22b$ **(C)** $42a+42b$
(D) $15a+66b$ **(E)** $18a+66b$

2. If a raindrop has a volume of 10 cubic millimeters, a certain school yard has dimensions 50 meters by 40 meters, and this yard receives 5 centimeters of rain, the number of raindrops that fell on the yard is

(A) 10^9 **(B)** 10^{10} **(C)** 10^{11} **(D)** 10^{12} **(E)** 10^{13}

3. Recall that $[x]$ denotes the greatest integer less than or equal to x. If $f(x) = [x^2] - [x]^2$, find $f(\pi)$.

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

4. How many subsets of $\{a,b,c,d,e\}$ have an odd number of elements?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

5. How many perfect squares are in the sequence of integers
$$1, 11, 111, 1111, 11111, 111111, \cdots$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

6. Suppose $a+b^{-1}=4$ and $b+a^{-1}=4/3$. If $a>b^{-1}$ what is the value of ab?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

7. If $f(3x+1) = \dfrac{2}{x+4}$, then $f(x+3) =$

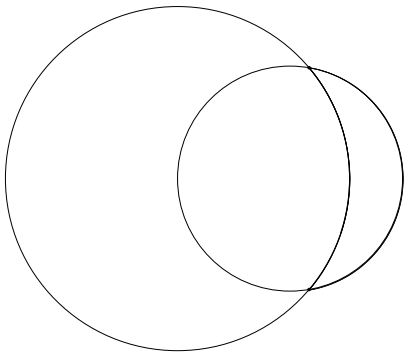
(A) $\dfrac{6}{x+14}$ **(B)** $\dfrac{6}{3x+14}$ **(C)** $\dfrac{6}{x+11}$ **(D)** $\dfrac{6}{x+17}$ **(E)** $\dfrac{2}{x+14}$

8. Independently of each other, Hezy and Zeke each randomly pick a real number between 0 and 3. What is the probability that their choices differ by at least 1?

(A) $1/9$ **(B)** $1/3$ **(C)** $1/2$ **(D)** $4/9$ **(E)** $5/9$

9. The distance between the two centers of the two circles is 3, the center of the larger circle lies on the smaller circle, and the points of intersection of the two circles lie on the same diameter of the smaller circle. Find the area of that part of the smaller circle that lies outside of the larger circle.

(A) 9 **(B)** $\pi/4$ **(C)** $9\pi/8 + 3\sqrt{2}/2$ **(D)** $3\pi - 3$ **(E)** 10



10. What is the positive difference between the largest and smallest real solutions of

$$x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. Let $f(x) = ax^2 + bx + c$, where a, b, and c are all non-zero constants. If $c = \dfrac{b^2}{4a}$, then the graph of f must

(A) be symmetric with respect to the y axis
(B) be symmetric with respect to the x axis
(C) be tangent to the x axis
(D) be tangent to the y axis
(E) have a maximum point

12. In acute triangle ABC, the intersection of its three altitudes, called the *orthocenter*, is labeled P. Given that $AP = 6$, $BP = 4$, and $BC = 10$, find AC.

(A) $\sqrt{120}$ **(B)** $\sqrt{130}$ **(C)** $\sqrt{140}$ **(D)** $\sqrt{150}$ **(E)** $\sqrt{160}$

13. A certain polynomial $P(x)$ has the property that all its coefficients are non-negative integers, none of which is larger than 6. If $P(7) = 2011$, what is P's coefficient of x^2?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. Square $ABCD$ of side-length 2 is inscribed in a circle. If chord AP bisects segment BC of the square, what is the square of the length of chord AP?

(A) 7 **(B)** 7.2 **(C)** 7.4 **(D)** 7.5 **(E)** 7.7

15. Five horses are in a race. In how many ways can they finish if ties are allowed?

(A) 511 **(B)** 530 **(C)** 531 **(D)** 541 **(E)** 625

16. Find the coefficient of x^{30} in the expansion of

$$(1 + x^3)(1 + x^6)(1 + x^9)(1 + x^{12}) \cdots (1 + x^{27})(1 + x^{30})$$

(A) 9 **(B)** 10 **(C)** 33 **(D)** 43 **(E)** 57

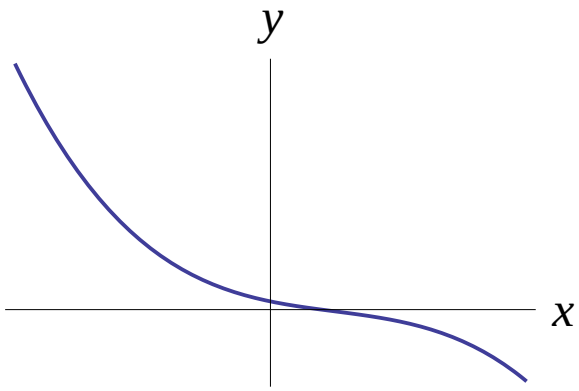
17. The graph of $y = ax^3 + bx^2 + cx + d$ is shown below. Which of the following must be true?

    I. $a < 0$

    II. $c < 0$

    III. $cd > 0$

    (A) I only **(B)** II only **(C)** I and II only
    (D) II and III only **(E)** I,II, and III



18. Find the radius of the sphere that contains the points $(0,-3,-2)$, $(0,-3,2)$, $(2,3,1)$, and $(-2,3,1)$.

    (A) $\frac{43}{12}$ **(B)** $\frac{\sqrt{1945}}{12}$ **(C)** $\frac{47}{12}$ **(D)** $\frac{\sqrt{2011}}{12}$ **(E)** $\frac{49}{12}$

19. How many solutions are there to the equation $(1.1)^x = \log_{1.1} x$?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

20. If $x^4 + \frac{1}{x^4} = 194$, then $x^6 + \frac{1}{x^6}$ could be equal to

    (A) 2702 **(B)** 2730 **(C)** 3088 **(D)** 3090 **(E)** 3102